National Technical and Vocational Qualification Framework

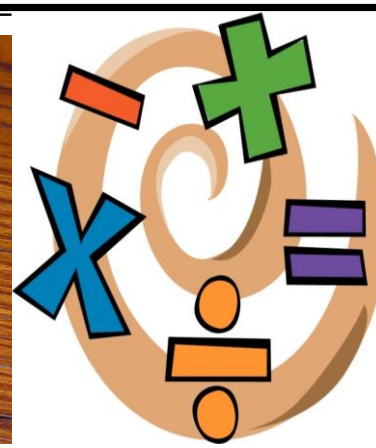
NTVQF

Competency-Based Learning Material

NTVQ Level 1

Using Basic Mathematical Concept





Bangladesh Technical Education Board

Agargoan, Shere Bangla Nagar

Dhaka-1207

TABLE OF CONTENTS

HOW TO USE THIS COMPETENCY-BASED LEARNING MATERIAL

Welcome to the module **Using Basic Mathematical Concept**. This module contains training materials and activities for you to complete.

This unit of competency, **"Use Basic Mathematical Concept"**, is one of the competencies of any NTVQ Level 1 Occupation, a course which comprises the knowledge, skills and attitudes required to become a Basic-Skilled Worker.

You are required to go through a series of learning activities in order to complete each learning outcome of the module. These activities may be completed as part of structured classroom activities or you may be required to work at your own pace. These activities will ask you to complete associated learning and practice activities in order to gain knowledge and skills you need to achieve the learning outcomes.

Refer to **Learning Activity Page** to know the sequence of learning tasks to undergo and the appropriate resources to use in each task. This page will serve as your road map towards the achievement of competence.

Read the **Information Sheets**. These will give you an understanding of the work, and why things are done the way they are. Once you have finished reading the Information sheets complete the questions in the Self-Check Sheets.

Self-Checks follow the Information Sheets in the learning guide. Completing the Self-checks will help you know how you are progressing. To know how you fared with the self-checks, review the **Answer Key**.

Complete all activities as directed in the **Job Sheets and/or Activity sheets**. This is where you will apply your new knowledge while developing new skills.

When working though this module always be aware of safety requirements. If you have questions, do not hesitate to ask your facilitator for assistance.

When you have completed all the tasks required in this learning guide, an assessment event will be scheduled to evaluate if you have achieved competency in the specified learning outcomes and are ready for the next task.

MODULE CONTENT

MODULE TITLE: **Using Basic Mathematical Concepts**

MODULE DESCRIPTOR:

This module covers the knowledge, skills and attitude needed to apply mathematical methods such as addition, subtraction, multiplication and division among others, in the routine tasks of an organization

NOMINAL DURATION: 40 hours

LEARNING OUTCOMES:

At the end of this module you MUST be able to:

1. Perform basic calculations involving whole numbers and decimals
2. Solve problems in fractions and percentages
3. Work with ratio and proportions
4. Use equations in calculating measurements of perimeter, areas and volume
5. Calculate equivalents measures involving different system of measurements
6. Interpret workplace data presented in graphs, charts and tables

ASSESSMENT CRITERIA:

1. Calculation requirements from workplace information are identified.
2. Appropriate mathematical methods are selected.
3. Mathematical language, symbols and terminology are used.
4. Appropriate units of measurement (such as kg, meter) and application may include measurement, volume, weight, density, percentage etc are understood.
5. Workplace information (project documents, graphs, charts, tables, spread sheets, item price quotations, equipment manuals) are interpreted.
6. Arithmetic processes to find solutions to simple mathematical problems are used.

LEARNING OUTCOME 1 — PERFORM BASIC CALCULATIONS INVOLVING WHOLE NUMBERS AND DECIMALS

CONTENTS:

1. Understand whole numbers
2. Perform four fundamental operations in whole numbers
3. Solve problems involving decimal numbers

ASSESSMENT CRITERIA:

1. Mathematical process in solving problems in whole numbers is used.
2. Calculation requirements in solving problems in decimals are identified.
3. Four fundamental operations are used in solving whole numbers and decimals.

CONDITIONS:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

LEARNING ACTIVITIES

LEARNING OUTCOME: Perform Basic Calculations Involving Whole Numbers and Decimals

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Addition and Subtraction of Whole Numbers	Read Information Sheet No. 1.1-1 on Addition and Subtraction of Whole numbers Answer Self Check 1.1 which is addition and subtraction of whole numbers then compare answers to Answer Key.
Multiplication and Division of Whole Numbers	Read Information Sheet No. 1.2 on Multiplication and Division of Whole numbers Answer Self Check 1.2 on Multiplication and Division of Whole numbers
Addition and Subtraction of Decimals	Read Information Sheet No. 1.3 on Addition and Subtraction of Decimals Answer Self Check 1.3
Multiplication and Division of Decimals	Read Information Sheet No. 1.4 on Multiplication and Division of Decimals Answer Self Check 1.4. Refer your answer to the Answer Key

INFORMATION SHEET 1.1-1

Addition and Subtraction of Whole Numbers

Learning Objectives:

After reading this Information Sheet, you should be able to add and subtract whole numbers.

INTRODUCTION TO WHOLE NUMBERS

Reading and Writing whole numbers

The **decimal system** of writing numbers uses the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to write any number. For example, these digits can be used to write the **whole numbers**:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and so on.

Each digit in a whole number has a place value. The following place value chart shows the names of the different places used most often.

Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

Example 1 *Identifying Whole Numbers*

(a) In the whole number 42, the 2 is in the ones place and has a value of two **ones**.

(b) in 29, the two is in the tens place and has a value of two **tens.**

(c) in 281, two is in the hundreds place and has a value of 2 **hundreds.**

The value of 2 in each number is different.

42 → 2 ones

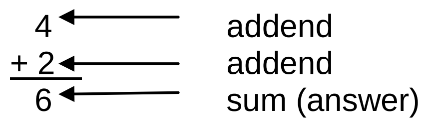
29 → 2 tens

281 → 2 hundreds

ADDITION OF WHOLE NUMBERS

The process of finding the total of two or more numbers is called *addition.*

In addition, the numbers being added are called ***addends,*** and the resulting answer is called ***sum*** or ***total.***



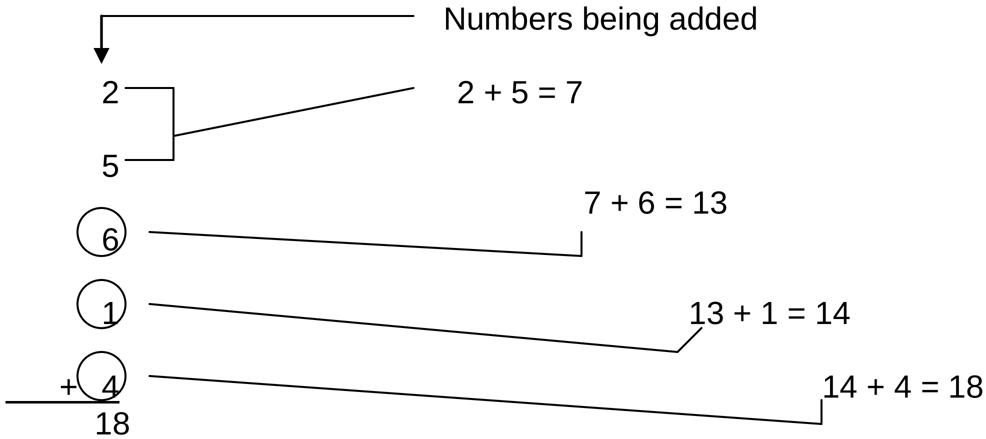
To change the order of the numbers in an addition problem without changing the sum, we use the **commutative property of addition.**

For example, the sum 3 + 8 is the same as the sum 8 + 3. This allows the addition of the same number in a different order.

To add several numbers, first write them in a column. Add the first number to the second. Add this sum the third digit, continue until all the digits are used.

Example 1. *Adding More Than Two Numbers*
Add 2, 5, 6, 1 and 4.

SOLUTION



If numbers have two or more digits, first arrange the numbers in columns so that the ones digits are in the same column, tens are in the same column, hundreds are in the same column, and so on. Next, add.

Example 2. *Adding without Carrying*
Add 410, 21, 106, and 52.

SOLUTION
First arrange the numbers in a column, with ones digits at the right.

hundreds in a column
tens in a column
ones in a column

```
    4 1 0
      2 1
    1 0 6
+     5 2
```

ones digits at the right

Start at the right and add the ones digits. Add the tens digits next, and finally, the hundreds digits.

```
    4 1 0
      2 1
    1 0 6
+     5 2
---------
    6 8 9
```

sum of ones
sum of tens
sum of hundreds

The sum of the four numbers is **689**.

If the sum of the digits in a column is more than 9, use **carrying.**

Example 3. Adding with Carrying
Add 47 and 35.

SOLUTION
Add ones.

```
    4 7
+   3 5
```

sum of ones is 12.

Since 12 is 1 ten plus 2 ones, place 2 in the ones column and carry 1 to the tens column.

```
     1
     4 7
+    3 5            7 + 5 =12
       2
```

Add in the tens column.

```
    1
    4 7
+   3 5
    8 2
    ↑          sum of the digits in tens column
```

Example 4. *Adding with Carrying*
Add 691, 3461, 32, 4, 17.

SOLUTION
Step 1 Add the digits in the ones column

```
         1 1
       6 9 1                                       carry to the tens
     3 4 6 1                                           column
         3 2     sum of the ones column is 15.
           4                                       write in the tens
+        1 7                                           column
           5
```

In 15, the 5 represents 5 ones and is written in the ones column, while 1 represents 1 tens and is carried to the tens column.

Step 2 Now add the digits in the tens column, including the carried 2.

```
       2 1                                         Carry to the hundreds
       6 9 1                                           column
     3 4 6 1
         3 2     sum of the tens column is 2 0.
           4
+        1 7                                       write in the
         0 5                                         tens column
```

Step 3
thousands

```
   1 2 1
     6 9 1
   3 4 6 1
       3 2
         4
+      1 7
   -------
     2 0 5
```

sum of the hundreds column is 1 2.

carry to the column

write in the hundreds column

Step 4

```
   1 2 1
     6 9 1
   3 4 6 1
       3 2
         4
+      1 7
   -------
   4 2 0 5
```

sum of the thousands column is 4.

write in the thousands column

Finally, 691 + 461 + 32 + 4 + 17 = 4205.

There are several words or phrases in English that indicate the operation of addition. Here are some examples.

added to	3 added to 5	5 + 3
more than	7 more than 5	5 + 7
the sum of	the sum of 3 and 9	3 + 9
increased by	4 increased by 6	4 + 6
the total of	the total of 8 and 3	8 + 3
plus	5 plus 10	5 + 10

SUBTRACTION OF WHOLE NUMBERS

Subtraction is the process of finding the difference between two numbers.

Suppose you have $50, and you spend $20 for a hamburger. You have then $30 left. There are two different ways of looking at these numbers:

As an addition problem:

$20 +$30 = $50

As a subtraction problem:

$\$50 - \$20 = \$30$

As this example shows, an addition problem can be changed to a subtraction problem and a subtraction problem can be changed to an addition problem.

In subtraction, as in addition, the numbers in a problem have names. In the example above, 50 – 20 = 30, the number 50 is the **minuend**, 20 is the **subtrahend**, and 30 is the **difference** or answer.

50 – 20 = 30

Minuend subtrahend difference

Subtract the two numbers by lining up the numbers, so the digits in the ones place are in the same column. Next, subtract by columns, starting at the right with the ones column.

Example 1. *Subtracting Two Numbers*

Ones digits are lined up in the same column.

(a)

```
  5 3
-  2 1
  3 2
```

3 – 1 = 2
5 – 2 = 3

(b)

```
  6 9 8 2
-     3 8 0
  6 6 0 2
```

2 – 0 = 2
8 – 8 = 0
9 – 3 = 6
6 – 0 = 6

If a digit in the subtrahend is larger than the digit in a minuend, **borrowing** will be necessary.

Example 2 *Subtracting with Borrowing*

(a) Subtract 18 from 63.

SOLUTION

$$\begin{array}{r} 63 \\ -\ 18 \\ \hline \end{array}$$

In the ones column, 8 is larger than 3, so borrow a 10 from the 6 (which represents 6 tens or 60).

60 – 10 = 50

$$\begin{array}{rr} 5 & 13 \\ \not{6} & 3 \\ -\ 1 & 8 \\ \hline \end{array}$$

10 + 3 = 13

Now subtract 8 from 13, and then 1 from 5.

$$\begin{array}{rr} 5 & 13 \\ \not{6} & 3 \\ -\ 1 & 8 \\ \hline 4 & 5 \end{array}$$

difference

Finally, 63 – 18 = 45. Check by adding 18 and 45.

(b) Subtract 378 from 692.

SOLUTION

hundred / tens / Ones	Hundre / tens / ones	hundred / tens / ones	hundred / tens / ones
$\begin{array}{rrr} & 8+1 & \\ 6 & \not{9} & 2 \\ -\ 3 & 7 & 8 \end{array}$	$\begin{array}{rrr} & 8+1 & 10 \\ 6 & \not{9} & 2 \\ -\ 3 & 7 & 8 \end{array}$	$\begin{array}{rrr} & 8 & 12 \\ 6 & \not{9} & \not{2} \\ -\ 3 & 7 & 8 \end{array}$	$\begin{array}{rrr} & 8 & 12 \\ 6 & \not{9} & \not{2} \\ -\ 3 & 7 & 8 \\ 3 & 1 & 4 \end{array}$
Because 8 > 2, borrowing is necessary. 9 tens = 8 tens + 1 ten.	Borrow one ten from the tens column and write ten in the ones' column	Add the borrowed 10 to 2	Subtract the digits in each column

The phrases below are used to indicate the operation of subtraction. An example is shown at the right side of each phrase.

minus	8 minus 5	8 - 5
less	9 less 3	9 - 3
less than	2 less than 7	7 - 2
the difference between	the difference between 8 and 2	8 - 2
decrease by	5 decrease by 1	5 – 1

Example 3. Find the difference between 5 6 2 and 1 9 4.

From the phrases that indicate subtraction, the difference between 562 and 194 can also be written as 562 – 194.

```
    5 12        4 15 12
  5 6 2         5  6  2
- 1 9 4       - 1  9  4
      8         3  6  8   difference
```

Example 3. *Borrowing with Zeroes*

Subtract.

```
  2 9 0 3
- 1 5 8 6
```

SOLUTION

It is not possible to borrow from the tens position. Instead, first borrow from the hundreds position.

```
900 – 100 =800     100 + 0 = 100
          ↘          ↓
             8  10
         2   9   0   3
      -  1   5   8   6
```

Now we may borrow from the tens position.

```
          9   ←  100 – 10 = 90
      8  10  13 ←  10 + 3 = 13
  2   9   0   3
- 1   5   8   6
```

Complete the problem.

```
          9
  1   8  10  13
  2   9   0   3
- 1   5   8   6
  1   3   1   7     difference
```

SELF-CHECK 1.1-1

1. Add the following whole numbers:
   a) 35 + 2 + 327 + 16
   b) 9 + 5 + 22
   c) 836 + 41
   d) 10 + 37 + 1
   e) 92 + 46

2. Subtract the following whole numbers:
   a) 769 less 252
   b) Subtract 74 from 328
   c) 63 minus 29
   d) 21 – 8
   e) 526 – 87

3. Solve for the following:
   a. There were 16 boys and 7 girls who came late during the flag raising ceremony. How many people were late in the ceremony?

   b. There were 25 trainees at the start of the training. Six of them did not finish the course. How many trainees graduated from the training?

ANSWER KEY 1.1-1

1. a) 380
   b) 36
   c) 877
   d) 48
   e) 138

2. a) 517
   b) 254
   c) 34
   d) 13
   e) 439

3. a) 23
   b) 19

INFORMATION SHEET 1.1-2

Multiplication and Division of Whole Numbers

Learning Objectives:

After reading this Information Sheet, you should be able to perform multiplication and division of whole numbers according to standard mathematical procedure.

MULTIPLICATION OF WHOLE NUMBERS

Adding the number 3 four times gives 12.

$$3 + 3 + 3 + 3 = 12$$

Multiplication is a shortcut for this repeated addition. The numbers being multiplied are called factors. The answer is called the product. For example, the product of 3 and 4 can be written with the symbol x, a raised dot, or parentheses, as follows.

3	factor (also called *multiplicand*)
X 4	factor (also called *multiplier*)
12	product

$3 \times 4 = 12$ $\quad 3 \cdot 4 = 12$ $\quad (3)\,(4) = 12$

The basic facts for multiplying one-digit numbers should be memorized. Multiplication of larger numbers requires the repeated use of the basic multiplication facts.

Example 1. *Carrying with Multiplication*
Multiply.

(a) $\begin{array}{r} 26 \\ \underline{\times \quad 8} \end{array}$

Start by multiplying in the ones column.

$\begin{array}{r} {}^{4} \\ 2\,6 \\ \underline{\times \quad 8} \\ 8 \end{array}$ $\quad 6 \times 8 = 48$ carry the 4 to the tens column
Place the 8 in the ones column

Next, multiply the 8 ones and the 2 tens.

```
   4
   2 6      2 x 8 = 16
X    8
     8
```

Add the 4 that was carried to the tens column.

```
   4
   2 6      16 + 4 = 20
X    8
   208
```

(b)

```
   7 2 4
X      5
```

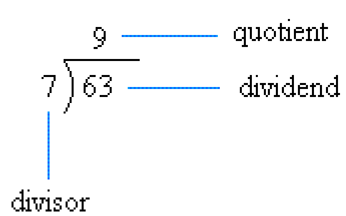
Work as shown.

```
   1 2
   7 2 4
X      5
 3 6 2 0    5 x 4 = 20   ones; write 0 ones and carry 2 tens.
            5 x 2 = 10   tens; add the 2 tens to get 12; write the 2 tens and carry 1 hundred
            5 x 7 = 35   hundreds; add the 1 hundred to get 36.
```

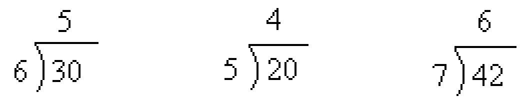
DIVISION OF WHOLE NUMBERS

Dividing whole numbers is the opposite of multiplying whole numbers. It is the process by which we try to find out how many times a number (divisor) is contained in another number (dividend).

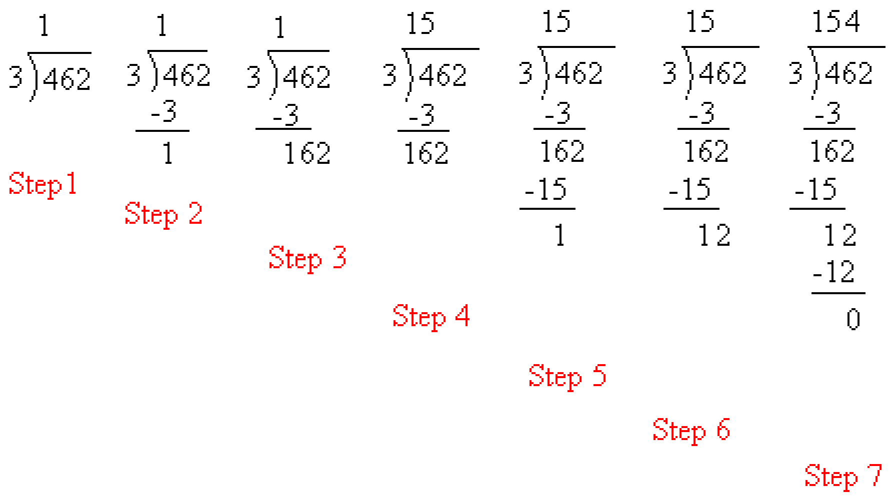
The answer in the division problem is called a quotient. In the division problem below (63 ÷ 7) , 7 is contained into 63, 9 times. (9 × 7 = 63)



Other examples:



When the dividend is bigger than 100, the answer may not be obvious. In this case you need to do long division. Study the following example (462 ÷ 3) carefully.



It is not easy to see immediately how many times 3 is contained into 462. It may not be easy also to see how many times 3 is contained into 46. However, it is fairly easy to see that 3 is contained into 4 once.

Therefore, we do this in step 1 and put the 1 above the 4.

In step 2, we multiply 1 by 3 and subtract the answer (3) from 4.

In step 3, we bring down the 62. Now, we need to find out how many times 3 is contained in 162. Still, it may not be obvious, so we will try to find out instead how many times 3 is contained into 16.

This is done in step 4 and we see that 3 is contained into 16, 5 times. We put the 5 above the 4.

In step 5 , we multiply 5 by 3 and subtract the answer (15) from 16.

In step 6 , we bring down the 2.

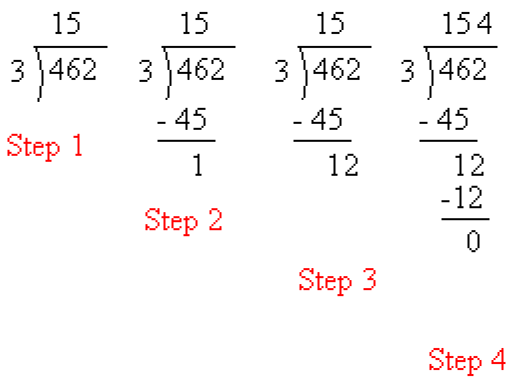
In step 7 , we try to find out how many times 3 is contained into 12.

3 × 4 = 12, so 3 is contained into 12, 4 times.

Finally, we put the 4 above the 6.

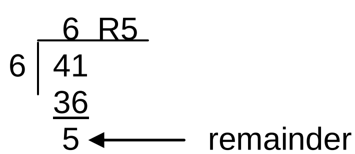
The answer is 154. or 3 is contained into 462, 154 times

The same division can be done faster if you can find out how many times 3 is contained into 45. 45 contains 3, 15 times. Then, you can finish the problem in 4 steps



When two numbers do not divide exactly, a number called the remainder is left over.

Example: Divide 41 by 6…



SELF-CHECK 1.1-2

I. Perform the indicated operations:

1. Find the product of 92 and 7.
2. Multiply 708 by 65.
3. 34 x 29
4. (102) (25)
5. How much is 18 times of 9?
6. 450 divide by 15
7. How many 26 are there in 208?
8. Divide 2,637 by 83, show the remainder
9. 820 ÷ 34
10. Find the quotient of 755 divided by 28

II. Solve the following:

1. There are 24 cans of soft drinks in 1 case. How many cans are there in 24 cases?
2. A kilo of meat cost $2.5, how much would be the cost of 5.5kg of meat?
3. A worker's salary is $11 per day. How much is weekly earnings if he works 6days in a week?
4. If 24 books cost $4,680, find the cost of each book.
5. Ana, Rose and George were paid $35 for a weekend work. Find out how much was their individual earning.

ANSWER KEY 1.1-2

I.

1. 644
2. 46,020
3. 986
4. 2,550
5. 162
6. 30
7. 8
8. 31 remainder 64
9. 24 remainder 4
10. 26 remainder 27

II.

1. 576
2. $13.75
3. $66
4. $195
5. $11.66

INFORMATION SHEET 1.1-3

Addition and Subtraction of Decimals

Learning Objectives:

After reading this Information Sheet, you should be able to understand and perform addition and subtraction of decimals.

Decimal System

Decimal system is used as another way to show parts of a whole. The decimal point is used to separate the whole-number part from the fractional part of the number.

Example: In the number **5.82**, the whole number part is **5** and the fractional part is **82.** The numbers at the right side of the decimal point is part of a whole; the value of which is less than one.

Addition of decimals

Step 1. Line up the decimal points.
Step 2. Next, add as with whole numbers.
Step 3. The decimal point in the answer appears directly below the decimal point in the problem

Example:

In adding 15.32 and 48.46:
Write the numbers vertically, with decimal points lined up.

```
    15 . 32
+   48 . 46
-----------
       ↑
       └──────── Decimal points are lined up
```

Add as with whole numbers, and place the decimal point in the answer under the decimal point in the problem.

```
    15 . 32
```

```
+    48 . 46
     63 . 78
        ↑
        └──────────  Decimal point in answer is under
                     decimal points in problem
```

If the numbers do not all have the same number of digits, using the following rule helps line up the digits to the right of the decimal point and makes addition easier.

Step 1. Find the number with the most digits after the decimal point.
Step 2. Attach zero to the right of the other number (to keep the places lined up), so that they all have the same number of digits after the decimal point.
Step 3. Next, add.
Example:

1. Add
   7.5 and 8.39

Solution:
There are two digits after the decimal point in 8.39, so attach one zero to the right in 7.5

```
     7 . 5 0  ←—— the zero is attached
+    8 . 3 9
    15 . 8 9
```

2. 4.28 + 11 + 3.735

```
     1  1
     4.2 8 0  ←—— one 0 is attached.
     9.0 0 0  ←—— 9 is a whole number; decimal point and three 0's are attached
+    3.7 3 5  ←—— no 0's are attached
    17.0 1 5
```

Notice in example 2 that 9 is really 9.,with the decimal point at the right. If no decimal point appears in a whole number, place one at the far right.

Subtraction of decimals

Subtraction of decimals is done in much the same way as subtraction of whole numbers. Use the following steps.

Step 1. Write the problem vertically with decimal points lines up in a column
Step 2. Bring the decimal point straight down.
Step 3. Subtract the numbers as if they were whole numbers. It may be necessary to use zeros as placeholders in one of the numbers.

Example:

Subtract each of the following:

a) 18.21 from 29.34

solution:

```
Step 1        29.34 ←  number subtracted from (minuend)
            - 18.21 ←  number being subtracted (subtrahend)
            -------

Step 2        29.34
            - 18.21
            -------
                .   ←  bring decimal point down

Step 3        29.34
            - 18.21
            -------
              11.13 ←  subtract as whole number (difference)
```

b) 56.78 from 143.25

Solution: borrowing is needed here.

```
     0 13 12   11 15
     1  4  3 .  2  5    (1, 4, 3, 2, 5 crossed out)
 -         5  6 . 7  8
 -------------------
           8  6 . 4  7
```

Check the answer by adding 86.47 and 56.78. The sum should be 143.25.

c) 18.6 from 32.418

Solution: use the same steps as above, remembering to attach zeros.

```
     32.4 18
 -   18.6 00 ←  Attach two 0's
 -----------
     24.8 18 ←  Next, subtract as usual
```

d) 5.96 from 15

Solution:

```
     15. 00 ←  Attach two 0's
 -    5. 96
 ----------
      9. 04 ←  subtract as usual
```

SELF-CHECK 1.1-3

I. Perform the indicated operation.
a) 6.54 + 9.8
b) 14 + 29.823 + 45.7
c) Subtract 36.7 from 58.9
d) 73.5 minus 19.2
e) 38.9 – 27.807

II. Solve the following:
a) Chris James worked at ACA Video 4.5 days one week, 6.25 days another week, and 3.74 days a third week. How many days did he work altogether?
b) At a bakery, Susie bought $7.42 worth of muffins, $10.09 worth of croissants, and $17.19 worth of cookies for a staff party. How much money did she spend altogether?
c) Justine drove on a five-day vacation trip. He drove 8.6 hours the first day, 3.7 hours the second day, 11.3 hours the third day, 2.9 hours the fourth day, and 14.6 hours the fifth day. How many hour did he drive?
d) Tom has agreed to work 42.5 hours at a certain job. He has already worked for 16.35 hours. How many more hours must he work?
e) A man buys $31.09 worth of sporting goods and pays with a $50 bill. How much change should he get?

ANSWER KEY 1.1-3

I. a) 16.34
   b) 89.523
   c) 22.2
   d) 54.3
   e) 11.093

II. a) 14.49 days
   b) $34.7
   c) 41.1 hours
   d) 26.15 hours
   e) $18.91

INFORMATION SHEET 1.1-4
Multiplication and Division of Decimals

Learning objectives:

After reading this Information Sheet, you should be able to perform multiplication and division of decimal.

Multiplication of Decimal

To multiply decimals, use the following steps:

Step 1. Multiply the numbers (factors) as if they were whole numbers. (it is not necessary to line up decimal points.)

Step 2. Find the total number of digits to the right of the decimal points in the numbers being multiplied.

Step 3. Position the decimal point in the answer by counting from the right to left the number of decimal places found above. It may be necessary to attach zeros to the left of the digits in the answer.

Examples:

a) Multiply 4.83 and 2.4

Solution:

Step 1. Multiply the numbers as if they were whole numbers.

```
     4.83
X     2.4
     1132
     966
    10792
```

Step 2. Count the number of digits to the right of the decimal points.

```
     4.83 ←   2 decimal places
X     2.4 ←   1 decimal place
     1132     3 ← total decimal places
     966
```

10792

Step 3. Count from the right in the answer over 3 places to position the decimal point.

```
      4.83
X      2.4
     1 132
       966
    10.792
      ↑_______________ decimal point – 3 decimal places from the right
```

b) Multiply .083 by .01.

Solution:
Start by multiplying as above.

```
     .083 ←  3 decimal places
X     .01 ←  2 decimal places
       83 ←  5 decimal places are in answer
```

The answer has only two digits, but five are needed. So attach three zeros at the left.

```
       83
 000  83 ← three zeros at left
.000  83 ← decimal point is at left
```

Division of decimals

There are two kinds of decimal division problems; those in which a decimal is divided by a whole number, and those in which a decimal is divided by a decimal.

1. Dividing a decimal by a whole number
   Divide a decimal by a whole number by placing a decimal point in the quotient (answer) directly above the decimal point in the dividend. Then divide as if both numbers were whole numbers.

Example:
   a) Divide 21.93 by 3

Solution
Place the decimal point directly above the decimal point in the dividend.

```
  ↓‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾|
  .              directly above
```

$$3\overline{)21.93}$$

Divide as if the numbers were whole numbers.

$$\begin{array}{r} 7.31 \\ 3\overline{)21.93} \end{array}$$

Check by multiplying the divisor 3 and the quotient 7.31. the product should be equal to the dividend 21.93.

b) Divide 1.5 by 8

Solution

$$\begin{array}{r} .1 \\ 8\overline{)1.5} \end{array}$$ start as above

Attach as many zeros after the 5 as necessary to make the quotient come out even, or until the desired accuracy is obtained.

$$\begin{array}{l} \quad .1\,875 \\ 8\overline{)1.5\,000} \leftarrow \text{three 0's are attached} \\ \quad \underline{8} \\ \quad 7\,0 \\ \quad \underline{6\,4} \\ \quad\;\; 6\,0 \\ \quad\;\; \underline{5\,6} \\ \quad\quad 4\,0 \\ \quad\quad \underline{4\,0} \\ \quad\quad\;\; 0 \leftarrow \text{comes out even (no remainder)} \end{array}$$

Note: Attach zero to the dividend until you reach a remainder of 0 or a repeating remainder. This does not change the value of the dividend.

2. Dividing a decimal by a decimal
   Divide a decimal by another decimal with the following rule:
   Step 1. To divide a decimal by another decimal, move the decimal point in the divisor to the right of the last digit.
   Step 2. Move the decimal point in the dividend as many places to the right until the decimal point is positioned right after the last digit of the dividend making it appear as if it is a whole number.
   Step 3. Move the decimal point in the divisor to the right as many places as were done in the dividend.
   Step 4. The decimal point in the quotient goes directly above the new position of the decimal point in the dividend. Then divide as usual.

Example:

a) Divide 27.69 by .3

Solution

.3.)27.6. 9 — Move the decimal points in the divisor and dividend place to the right.

92. 3
3)276. 9 — Place the decimal point in the quotient and divide

Check using the original numbers.

.3 x 92.3 = 27.69

SELF-CHECK 1.1-4

I. Multiply the following:

a) 11.62 x 4.01
b) 1.02 x 0.08
c) .0081 x .007
d) 146.8 x 3.4
e) .3 x 12.5

II. Divide the following:

a) 28.82 ÷ .2
b) 9.0064 ÷ .8
c) 32 ÷ .005
d) 70 ÷ 2.8
e) 80 ÷ .05

ANSWER KEY 1.1-4

I.
a) 46.5962
b) 0.0816
c) 0.0000567
d) 499.12
e) 3.75

II.
a) 144.10
b) 11.258
c) 6400
d) 25
e) 1600

LEARNING OUTCOME 2 SOLVE PROBLEMS IN FRACTIONS AND PERCENTAGES

CONTENTS:

1. Understand Fractions and Percentages
2. Perform four fundamental operations in Fractions
3. Solve problems involving Percentages

ASSESSMENT CRITERIA:

1. Mathematical process in solving problems in fractions is used.
2. Calculation requirements in solving problems in percentages are identified.
3. Four fundamental operations in solving problems in fractions are used.
4. Mathematical equations are used in solving problems in percentages.

CONDITIONS:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

Learning Activities

LEARNING OUTCOME: Solve Problems in Fractions and Percentages

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Understanding Fractions	Read Information Sheet No. 1.2-1 – Understanding Fractions Answer Self Check 2.1
Addition and Subtraction of Fractions	Read Information Sheet No. 2.2 on Addition and Subtraction of Fractions Answer Self Check 2.2. Refer to Answer Key
Multiplication and Division of Fractions	Read Information Sheet No. 2.3 on Multiplication and Division of Fractions Answer Self Check 2.3. Compare your answers to Answer Key
Understanding Percent and Percentages	Read Information Sheet No. 2.4 on Percent and Percentages Answer Self Check 2.4. Refer your answer to Answer Key.

INFORMATION SHEET 1.2-1

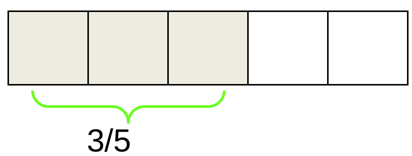
Understanding Fractions

Learning Objectives:

After reading this Information Sheet, you should be able to understand fractions and its kinds. You should also be able to write improper fractions in a form of mixed number and vise versa.

Information sheet no. 1 discussed whole numbers. Most of the time parts of whole numbers are being considered. One way to write parts of a whole is with decimal and the other way is with **fractions**.

Fractions are used to represent a portion of a whole which consist of equally divided parts.



The figure above has 5 equal parts. The 3 shaded parts are represented by the fraction 3/5 which is read as “three fifth.”

In the fraction ¼, the number 1 is the numerator, and 4 is the denominator. The denominator of a fraction shows the number of equivalent parts in the whole.

There are two kinds of fraction, a proper and an improper fraction. If the numerator of a fraction is *smaller* than the denominator, the fraction is a **proper fraction**. If on the other hand the numerator is *greater than or equal* to the denominator, the fraction is an **improper fraction.**

proper fractions
1/4, 1/3, 3/5, 2/3
Less than one

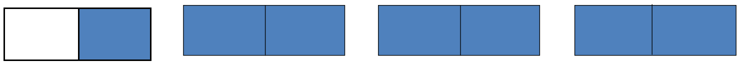
improper fractions
8/3, 5/2, 7/4, 6/6
more than 1 — equal to one

A proper fraction has a value which is less than one, while an improper fraction has a value that is 1 or greater.

In some cases, a fraction and a whole number come together. This number is called a mixed number. For example, the mixed number

3 1/2 represents 3 + 1/2,

or 3 wholes and 1/2 of a whole, read 3 1/2 as “three and one half”.



Illustration

In this figure, the mixed number 3 1/2 is equal to the improper fraction 7/2. This means that every mixed number has an equivalent improper fraction and vise versa.

Writing mixed number as an improper fraction

Step 1. **Multiply** the denominator of a fraction and whole number.
Step 2. **Add** to this product the numerator of the fraction.
Step 3. Write the result of step 2 as the **numerator** and the original denominator as the **denominator**.

Example

Write 7 2/3 as an improper fraction (numerator greater than the denominator).

Solution
Step 1. 7 2/3 7 x 3 = 21 multiply 7 and 3

Step 2 7 2/3 21 + 2 = 23 Add 2

Step 3 7 2/3 = 23/3 Same denominator

Changing improper fraction as a mixed number

Write an improper fraction as a mixed number by dividing the numerator by the denominator. The quotient is the whole number (of the mixed number), the remainder is the numerator of the fraction part, and the denominator remains unchanged.

Example
Write as mixed numbers.

a) $15/6$

Solution
Divide 15 by 6.

$$\begin{array}{r} 2 \\ 6\overline{)15} \\ \underline{12} \\ 3 \end{array} \leftarrow \text{remainder}$$

the quotient 2 is the whole number of the mixed number. The remainder three is the numerator of the fraction and the denominator remains as 5.

15/6 = 2 3/6

Same denominator

b) 32/7

Solution

Divide 32 by 7.

$$7\overline{)32} = 4 \text{ remainder } 4 \quad (28)$$

32/7 = 4 4/7

SELF-CHECK 1.2-1

I. True or False. Write true if the statement if true and False if otherwise.

________ 1. Fractions are used to represent a portion of a whole which consist of equally divided parts.

________ 2.The numerator of a fraction shows the number of equivalent parts in the whole.

________ 3.If the numerator is greater than or equal to the denominator, the fraction is a proper fraction**.**

________ 4. An improper fraction has a value which is less than one.

________ 5. Every mixed number has an equivalent improper fraction and vise versa.

II. Convert the following:

1. Improper Fraction to mixed number

   a) $\frac{28}{3}$

   b) $\frac{97}{26}$

   c) $\frac{19}{5}$

2. Mixed number to improper fraction

   a) $17\,\frac{3}{8}$

   b) $5\,\frac{4}{7}$

   c) $9\,\frac{7}{12}$

ANSWER KEY 1.2-1

I. True or False

a) True
b) False
c) False
d) False
e) True

II. 1.a. $9\frac{1}{3}$

1.b. $3\frac{19}{26}$

1.c. $3\frac{4}{5}$

2.a. $\frac{139}{8}$

2.b. $\frac{39}{7}$

2.c. $\frac{115}{12}$

INFORMATION SHEET 1.2-2

Multiplication and Division of Fraction

Learning Objectives:

After reading this Information Sheet, you should be able to perform multiplication and Division of fractions.

Multiplication of Fraction

Multiplication of fraction follows a very simple rule. To multiply two or more fractions, just multiply the numerators to get the numerator of the product; and multiply the denominators for the denominator of the product.

Example 1

If the fractions 2/3 and 1/4 are to be multiplied, do it as follows

denominators

$$2 / 3 \quad x \quad 1/4 \quad = \quad \frac{2 \times 1}{3 \times 4} \quad = \quad \frac{2}{12}$$

numerators

then reduce the answer to the lowest term. In this case, 2/12 when reduced to lowest term will be 1/6.

In reducing fraction to lowest term, the simplest way is finding a number to which both the numerator and the denominator can be divided by. In the above example, 2/12 is reduced to a similar fraction of 1/6. The fraction 1/6 was the result of dividing both the numerator 2 and the denominator 12 by 2. Fractions in lowest term have the same value as with the original fraction. Take for example the following fractions.

8/16 = 4/8 = 2/4 =1/2

The fractions above are similar fractions. The lowest term of 8/16 is ½. This came from dividing both the numerator 8 and the denominator 16 by 8. Notice that 8 is the highest number to which both the numerator and the denominator can be divided. In this case, we call 8 as a **factor** of both 8 and 16. In order to arrive at the lowest term of the fraction, you need to get the highest factor.

In the above example, 4 is also a factor of 8 and 16. This means that both the numerator 8 and the denominator 16 can be divided exactly by 4. By doing so, we will come up with the fraction 2/4. But this fraction is not yet in lowest term since there is still a number to which the numerator 2 and the denominator 4 can be divided into. So always find the largest factor in getting the lowest term of a fraction.

Example 2

Multiply 5/8 and 3/5 and reduce the answer to lowest term.

Solution

Multiply the numerators 5 and 3
Multiply the denominators 8 and 5

$$\frac{5 \times 3}{8 \times 5} = \frac{15}{40}$$

Reduce the answer to lowest term

$$\frac{15 \div 5}{40 \div 5} = \frac{3}{8}$$

In the example above, 5 is the largest number to which the numerator 15 and the denominator 40 can be divided into.

Example 3
Multiply 3/7 and 4 2/3

Solution

The above example involves a fraction and a mixed number. Convert first the mixed number to improper fraction.

3/7 x 4 2/3   =   3/7 x 14/3

Multiply as above

$$\frac{3 \times 14}{7 \times 3} = \frac{42}{21}$$

The answer is an improper fraction; convert to mixed number by dividing the numerator by the denominator

$$\frac{42}{21} = \quad 21\overline{)42} = 2, \quad 42 - 42 = 0$$

So,

3/7 x 4 2/3   = 2.

Example 4

Multiply 2/5 by 8.

Solution

To multiply a fraction and a whole number, write the whole number as a fraction with a denominator of 1.

2/5 x 8/1 ⟵ one being the denominator of the whole number 8

Multiply as above

$$\frac{2 \times 8}{5 \times 1} = \frac{16}{5}$$

The answer is an improper fraction so convert to mixed number

$$\frac{16}{5} = \begin{array}{r} 3 \\ 5\overline{)16} \\ \underline{15} \\ 1 \end{array} = 3\ 1/5$$

Division of Fractions

As discussed earlier, division is like asking how many of the divisors are there in the dividend. In the division problem 12 ÷ 3, it is asked many 3's are there in 12. In the same way, the division problem 2/3 ÷ 1/6 asks how many 1/6's are there in 2/3. Look at the figure.

Illustration

The figure shows that there are 4 of the 1/6's in 2/3, or

2/3 ÷ 1/6 = 4

The same answer can be obtained by using the following steps.

$$\frac{2}{3} \times \frac{6}{1} = \frac{12}{4} = 4$$

1/6 is inverted to get 6/1, then proceeded to multiplication.

Note: In dividing two fractions, multiply the dividend (first fraction) by the reciprocal (inverted form) of the divisor (second fraction).

Example 1

Divide 5/8 by 3/4 and reduce to lowest term

Solution

Invert 3/4 to get 4/3

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{5 \times 4}{8 \times 3} = \frac{20}{24} = \frac{5}{6}$$

Example 2

7 ÷ 1/3

Solution

Change 7 to 7/1. Next, invert 1/3 and multiply.

$$7 \div \frac{1}{3} = \frac{7}{1} \times \frac{3}{1} = \frac{7 \times 3}{1 \times 1} = \frac{21}{1} = 21$$

SELF-CHECK 1.2-2

Perform the indicated operation:

1. Multiply the following:

   a) $\frac{4}{7} \times \frac{3}{5}$

   b) 5 x $\frac{3}{9}$ x $\frac{6}{10}$

   c) 12 ½ x 2 ¾

   d) 1 ¼ x ¼

   e) 6 x 2 ½

2. Divide the following:

   a) $\frac{4}{5} \div \frac{6}{25}$

   b) $12 \div 2\frac{2}{3}$

   c) $\frac{3}{5} \div 5\ \frac{1}{4}$

   d) $7\frac{3}{8} \div 4$

   e) $2\frac{1}{3} \div 3\ \frac{4}{9}$

ANSWER KEY 1.2-2

I. Multiplication of fractions

a) $\frac{12}{35}$

b) $\frac{90}{90}$ or 1

c) $\frac{275}{8}$ or 34 $\frac{3}{8}$

d) $\frac{5}{16}$

e) $\frac{30}{2}$ or 15

II. Division of Fraction

a) $\frac{100}{30}$ or 3 $\frac{1}{3}$

b) $\frac{36}{8}$ or 4 ½

c) $\frac{12}{105}$

d) $\frac{59}{32}$ or 1 $\frac{27}{32}$

e) $\frac{63}{93}$ or $\frac{21}{31}$

INFORMATION SHEET 1.2-3

Addition and Subtraction of Fraction

Learning Objectives:

After reading this Information Sheet, you should be able to perform addition and subtraction of fractions.

Fractions with the same denominators are **like fractions.** Fractions with different denominators on the other hand are unlike fractions.

Identifying like and unlike fractions

Example:

a) 3/4, 1/4, 5/4, 6/4 and 4/4 are **like** fractions (all denominators are the same)
b) 7/12 and 2/5 are **unlike** fractions (different denominators)

Adding like fractions

Add like fractions as follows:

- The numerator of the answer (the sum) is found by adding the numerators
- The denominator of the sum is the denominator of the fractions.
- Always write the answer in lowest terms.

Example:

a) $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ ← add numerators ← same denominator

b) $\frac{1}{12} + \frac{7}{12} + \frac{1}{12} = \frac{1+7+1}{12} = \frac{9}{12} = \frac{3}{4}$

c) 2 ¼ + 3 ¾ + 5 ¼

Convert first mixed numbers to improper fractions

2 ¼ = 9/4

3 ¾ = 15/4

5 ¼ = 21/4

Rewriting the equation;

$$\frac{9}{4} + \frac{15}{4} + \frac{21}{4} = \frac{45}{4}$$

← sum of the numerators
← same denominator

Because the answer is an improper fraction, change it to mixed number.

$$\frac{45}{4} = 11\tfrac{1}{4}$$

Subtracting like fractions

To subtract like fractions:

- The numerator of the answer (the difference) is found by subtracting the numerators.
- The denominator of the difference is the denominator of the like fractions.
- Always write the answer in lowest terms.

Example:

a) $$\frac{11}{12} - \frac{7}{12} = \frac{11 - 7}{12}$$

← subtract numerators
← denominator of like fraction

$= 4/12$

Write in lowest terms.

$$\frac{11}{12} - \frac{7}{12} = \frac{4}{12} = \frac{1}{3}$$

b) $$\frac{8}{11} - \frac{3}{11} = \frac{5}{11}$$

c) $3\tfrac{1}{4} - 2\tfrac{3}{4}$

Convert first mixed numbers to improper fractions

$$3\tfrac{1}{4} = \frac{13}{4}$$

$$2\tfrac{3}{4} = \frac{11}{4}$$

$$\frac{13}{4} - \frac{11}{4} = \frac{2}{4}$$

Reduce to lowest term

2 = 1

4 2

Only like fractions can be added or subtracted. Because of this, *unlike* fractions must be rewritten as like fractions before adding or subtracting.

Do this with the least common multiple of the denominators.

The **least common multiple** (**LCM)** of two whole numbers is the smallest whole number divisible by both those numbers.

Finding the multiple of a number
The list shows multiple of 6.

6, 12, 18, 24, 30….

(The three dots show the list continues in the same pattern without stopping.) The next list shows multiple of 9.

9, 18, 27, 36, 45…

The smallest number in both lists is 18, so 18 is the least common multiple of 6 and 9; the number 18 is the smallest whole number divisible by both 6 and 9.

Multiple of 6	6, 12, 18, 24, 30,…
Multiple of 9	9, 18, 27, 36, 45,…

18 is the smallest number in the lists.

The least common multiple often must be used as a denominator for a list of fractions.

Writing a fraction with an indicated denominator

1. Find a numerator so that

$$\frac{2}{3} = \frac{\_\_}{15}$$

To find the new numerator, first divide 15 by 3.

$$\frac{2}{3} = \frac{\_\_}{15} \qquad 15 \div 3 = 5$$

Then multiply the product by the numerator of the original fraction

5 x 2 = 10

$$\frac{2}{} = \frac{10}{} \qquad 15 \div 3 = 5$$

3 15

10 will be the new denominator if 2/3 will be written with a denominator of 15.

2. Write the following fraction with the indicated denominator.

$$\frac{3}{4} = \frac{\_\_}{28}$$

Divide 28 by 4, getting 7. Now multiply 7 to the numerator 3 getting 21.

7 x 3 = 21

$$\frac{3}{4} = \frac{21}{28}$$

28 ÷ 4 = 7

21 will be the new denominator if 3/4 will be written with a denominator of 28.

Adding and Subtracting unlike fractions

Unlike fractions can be added and subtracted by first changing them to like fractions. Once the fractions are rewritten as like fractions, you can now add and subtract following the steps in dealing with like fractions.

Example 1

Add 2/3 and 1/9.

Solution

Since the least common multiple of 3 and 9 is 9, write the fractions as like fractions with a denominator of 9. The denominator is called the least common denominator of 3 and 9. First,

$$\frac{2}{3} = \frac{\_\_}{9}$$

Divide 9 by 3 getting 3. Next multiply the answer 3 by the numerator 2 getting 6.

$$\frac{2}{3} = \frac{6}{9}$$

Next, add the like fraction 6/9 and 1/9

$\underline{6}$ + $\underline{1}$ = $\underline{7}$

$$9 \quad 9 \quad 9$$

Example 2.

Add 3/5 and 1/4.

Solution

Since the least common multiple of 4 and 5 is 20, write the fractions as like fractions with a denominator of 20. The least common denominator of 4 and 5 is 20. First,

$$\frac{3}{5} = \frac{\quad}{20} \quad \text{and} \quad \frac{1}{4} = \frac{\quad}{20}$$

Divide 20 by 5 getting 4 then multiply by 3 getting 12.

$$\frac{3}{5} = \frac{12}{20}$$

Do the same with the other fraction. Divide 20 by 4 getting 5, then multiply by 1 getting 5.

$$\frac{1}{4} = \frac{5}{20}$$

Next, add the like fraction 12/20 and 5/20

$$\frac{12}{20} + \frac{5}{20} = \frac{17}{20}$$

The next example shows subtraction of unlike fractions.

Example 3

Subtract the following fractions:

a) $\frac{3}{4} - \frac{3}{8}$

b) $\frac{3}{4} - \frac{5}{9}$

Solution

As with addition, rewrite unlike fractions with a least common denominator.

a) $\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{3}{8}$

b) $\frac{3}{4} - \frac{5}{9} = \frac{27}{36} - \frac{20}{36} = \frac{7}{36}$

SELF- CHECK 1.2-3

Perform the indicated operation:

1. Add the following:

   a) $\frac{4}{5} + \frac{3}{5}$

   b) $5 + \frac{3}{10} + \frac{7}{10}$

   c) 12 ½ + 2 ¾

   d) $1\frac{3}{5} + 3\frac{2}{3}$

   e) $6 + 2\frac{1}{4} + \frac{5}{6}$

2. Subtract the following:

   a) $\frac{14}{25} - \frac{6}{25}$

   b) $12 - 2\frac{2}{3}$

   c) $5\frac{3}{5} - \frac{1}{4}$

   d) $7\frac{3}{8} - 4\frac{3}{5}$

   e) $12\frac{1}{3} - 3\frac{4}{9}$

ANSWER KEY 1.2-3

I. a) $\frac{7}{5}$

b) 6

c) $15\ \frac{1}{4}$

d) $5\ \frac{4}{15}$

e) $9\ \frac{2}{24}$ or $9\ \frac{1}{12}$

II. a) $\frac{8}{25}$

b) $9\ \frac{1}{3}$

c) $5\ \frac{7}{20}$

d) $2\ \frac{31}{40}$

e) $8\ \frac{8}{9}$

INFORMATION SHEET 1.2-4

Percent and Percentages

Learning Objectives:

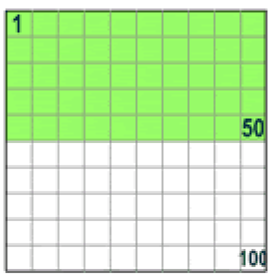
After reading this Information Sheet, you should be able to understand percent and percentages and solve problems involving percent.

Percent

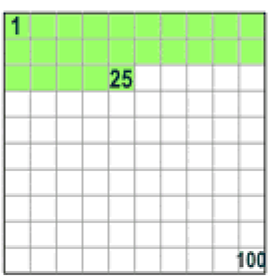
After understanding the use of four fundamental operations in fraction, it will be known that percent is equivalent to fraction.

A percent of a number is a method of expressing some part of whole numbers with a base of 100. Thus, 100% of a number is all of it.

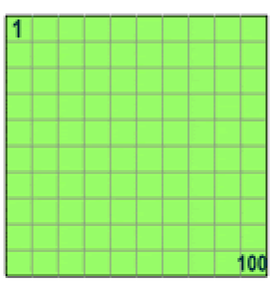
When you say “percent” you are really saying “per 100”



So 50% means 50 per 100 (50% of this box is green)



and 25% means 25 per 100 (25% of this box is green)



100% means all.

When you say 10 out of 100, it is $\frac{10}{100}$ or 10 percent or written as 10%.

The same as with:

$$15 \text{ of } 100 = \frac{15}{100} = 15\%$$

$$20 \text{ of } 100 = \frac{20}{100} = 20\%$$

$$50 \text{ of } 100 = \frac{50}{100} = 50\%$$

$$100 \text{ of } 100 = \frac{100}{100} = 100\%$$

It is said that percent is equivalent to fraction. So it means that every percent has an equivalent fraction and vise versa.

For example, to express ½, ¼, and ¾ as percents,

1/2	=	.50	=	50%
1/4	=	.25	=	25 %
3/4	=	.75	=	75%

Note that the fraction is changed to a decimal by dividing the numerator by the denominator first. Then the decimal point is moved two places right, dropped, and the percent sign is added.

Percentage

A percentage is another way of expressing a part as a fraction of a whole unit. All percentage problems consist of three elements: (a) the base, (b) the rate and (c) the percentage.

Base – the whole unit on which the rate operates
Rate – the number of hundredths parts taken. This is the number followed by the percent sign
Percentage – the part of the base determined by the rate

Given the problem;

5 % of 100 is 5, the base is 100, the rate is 5%, and the percentage is 5.

There are three cases that usually arise in dealing with percentage.

Case 1. To find the percentage when the base and rate are known.
Example 1: what number is 6% of 50?

The "of" means to multiply. Thus, to find the percentage, multiply the base by the rate. Of course the rate must be changed from a percent to a decimal before multiplying can be done. Rate times base equals percentage.

Thus,
6% in decimal = .06
.06 x 50 = 3
The number that is 6% of 50 is 3.

Example 2. what is 16% of 1400?

First convert 16% to its decimal form;
16% = .16
.16 x 1400 = 224
16% of 1400 is 224

To explain the case II and III, we notice in example 1 that the base corresponds to the multiplicand, the rate corresponds to the multiplier, and the percentage corresponds to the product.

50 (base or multiplicand)
.06 (rate or multiplier)
1.00(percentage or product)

Recalling that the product divided by one of its factors gives the other factor; we can solve the following problem:

Example 3. ?% of 60 = 20

We are given the base (60) and percentage (20).

60 (base)
? (rate)
20 (percentage)

We then divide the product (percentage) by the multiplicand (base) to get the other factor (rate). Percentage divided by base equals rate. The rate is found as follows:

$$\frac{20}{60} = .3333 = 33.33\%$$

Example 4. 45 is what percent of 960?

We are given the percentage 45 and base 960. So we are looking for the rate.
As with the example above, rate can be computed by dividing the percentage by base.

$$\frac{45}{960} = 0.0469 = 4.69\%$$

0.0469 is converted to percent by moving the decimal point to the right by two places, or multiplying by 100.

The rule for case ll, as illustrated in the foregoing problem, is as follows: To find the rate when the percentage and base are known, divide the percentage by the base. Write the quotient in the decimal form first, and finally as a percent.

Case lll

The unknown factor in case lll is the base, and the rate and percentage are known.

Example 5. 35 is 25% of what number?

? (base)
<u>.25</u> (rate)
35 (percentage)

We divide the product by its known factor to find the other factor. Percentage divided by rate equals base. Thus,

$$\frac{35}{.25} = 140 \text{ (base)}$$

Example 6. 16% of ? = 240

rate = 16%; percentage = 240; base = ?

$$\frac{240}{.16} = 1500 \text{ (base)}$$

The rule for case lll may be stated as follows. To find the base when the rate and percentage are known, divide the percentage by the rate.

SELF-CHECK 1.2-4

Solve for the following problems:

1. What is 8% of 100?
2. What percent of 75 is 24?
3. 27 is 30% of what number?
4. In the last election, 64% of the eligible people actually voted. If there were 325 voters, how many people were eligible?
5. An instructor's salary is $430. He receives a 9% increase last month. What is his new salary?

ANSWER KEY 1.2-4

1. 8
2. 32%
3. 90
4. 208
5. $468.7

LEARNING OUTCOME 3 WORK WITH RATIO AND PROPORTIONS

CONTENTS:

1. Understand Ratio and Proportions
2. Finding missing terms in a proportion
3. Solve problems in Ratio and Proportions

ASSESSMENT CRITERIA:

1. The concepts about ratio and proportion are understood.
2. Calculation requirements in solving problems in ratio and proportions are identified.
3. Mathematical equations in solving problems in ratio and proportion are used.

CONDITIONS:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

LEARNING ACTIVITIES

LEARNING OUTCOME: Work With Ratio and Proportions

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Understanding Ratio and Proportions	Read Information Sheet No. 1.3-1 – Ratio and Proportions Answer Self Check 1.3-1. Refer your answer to Answer Check.

INFORMATION SHEET 1.3-1

Ratio and Proportion

Learning Objectives:

After reading this Information Sheet, you should be able to solve problems in ratio and proportion.

Ratio

When comparisons are made between two quantities with the same units such as feet, inches, shillings, pounds, etc., this is known as ratio. Numbers may be related to each other in various ways and the number of times that one number may be contained in another number is known as ratio.

A ratio is a relationship between two numbers or like quantities. It can be written in three different ways.

The ratio of the number *a* to the number *b* is written as

a to ***b*** ***or*** ***a:b*** ***or*** ***a/b***

Writing a Ratio

The most common way of writing a ratio is in fraction form.

Example 1

7 bananas to 15 bananas

The ratio is $\frac{\text{7 bananas}}{\text{15 bananas}}$ $\frac{7}{15}$ ← 7 mentioned first, ← 15 mentioned second

Note that the number mentioned first always goes on top, and the units are not written because they cancel each other as do numerators and denominators of fractions.

Example 2

The width of a rectangle is 4ft and the length is 7ft. Write the ratio of width to length.

The ratio is $\frac{4}{7}$

Example 3

2 ½ days to 4 ¾ days

To write it as ratio

$$\frac{2\frac{1}{2}}{4\frac{3}{4}}$$

Convert mixed number to improper fraction

$$\frac{2\frac{1}{2}}{4\frac{3}{4}} = \frac{5/2}{19/4}$$

$$\frac{\frac{5}{2}}{\frac{19}{4}} = \frac{5}{2} \times \frac{4}{19} = \frac{20}{38} = \frac{10}{19}$$

Proportion

A proportion is a special form of an algebra equation. It is used to compare two ratios or make equivalent fractions.

$$\frac{1}{2} = \frac{3}{6}$$

The four parts of the proportion are separated into two groups, the **means** and the **extremes**, based on their arrangement in the proportion.

Extremes are read from left to right and top to bottom, the very first number and the very last number. This can be remembered because they are at the extreme beginning and the extreme end.

Means are the second and third numbers read from left to right and top to bottom. Remembering that mean is a type of average may help you remember that the means of a proportion are in the middle when reading left-to-right, top-to-bottom.

$$\frac{1}{2} = \frac{3}{6}$$

Numbers 1 and 6 are the extremes while 3 and 2 are the means.

Algebra properties tell us that the product of the means is equal to the product of the extremes.

In the above illustration, the product of 1 and 6, the extremes, is equal to the product of 2 and 3, the means.

$$1 \times 6 = 2 \times 3$$
$$6 = 6$$

Sometimes you may encounter a proportion that has one of its means or extremes missing, or uses another symbol such as a question mark you can treat it as if it was a variable. Or you can replace the question mark or blank space with a variable such as x.

$$\frac{9}{5} = \frac{90}{?}$$

$$\frac{9}{5} = \frac{90}{x}$$

In order to get the missing mean, divide the product of the extremes by one of the means. The same goes when one of the extremes is missing. Divide the product of the means by one of the extreme.

From the above illustration, the product of 9 and x, the extremes, is equal to the product of 5 and 90, the means.

Finding the Missing Term

Example:

1. $\frac{x}{4} = \frac{7}{8}$

One of the extremes is missing, in order to get the missing extreme, divide the product of the means (4 and 7) by one of the extremes (8).

$4 \times 7 = 28$ ← product of the means

$28 \div 8 = 3.5$ ← missing extreme

(↑ 8: One of the extremes)

2. $\frac{6}{x} = \frac{24}{32}$

One of the means is missing, in order to get the missing mean, divide the product of the extremes (6 and 32) by one of the means (24).

$6 \times 32 = 192$ ← product of the extremes

$192 \div 24 = 8$ ← missing mean

↑ one of the means (24)

3. If Jane can bake 50 hotcakes in 20 minutes, how many hotcakes can she bake in hour?

Write the figures in ratio:

$$\frac{50 \text{ hotcakes}}{20 \text{ minutes}} = \frac{x \text{ hotcakes}}{1 \text{ hour}}$$

In a proportion, the units should be the same. If the given figures are expressed in different units, convert to similar units.

1 hour = 60 minutes

$$\frac{50 \text{ hotcakes}}{20 \text{ minutes}} = \frac{x \text{ hotcakes}}{60 \text{ minutes}}$$

$$\frac{50}{20} = \frac{x}{60}$$

One of the means is missing, in order to get the missing mean, divide the product of 50 and 60, the extremes, by 20, one of the means.

$50 \times 60 = 3{,}000$

$3{,}000 \div 20 = 150$

A total of 150 hotcakes will be made for 1 hour.

SELF-CHECK 1.3-1

I. Write true of the statement is true and false if otherwise

_________ 1. Ratio is a relationship between two fractions.
_________ 2. Two similar ratios are proportion
_________ 3. Means are for ratio, extremes are for proportion
_________ 4. If the product of the extremes is equal to the product of the means, then it is a proportion.
_________ 5. Ratios are always expressed as fraction.

II. Find the missing term:

1. $\frac{?}{15} = \frac{9}{18}$

2. $\frac{5}{9} = \frac{90}{?}$

3. $\frac{12}{?} = \frac{30}{45}$

4. 6 Magazines cost $15. Find the cost of 14 magazines.

5. If 5 ounces of medicine must be mixed with 11 ounces of water, how many ounces of medicine would be mixed with 99 ounces of water?

ANSWER KEY 1.3-1

I. True or False

1. False
2. True
3. False
4. True
5. False

II. Finding the missing term

1. 7.5
2. 162
3. 18
4. $35
5. 45

LEARNING OUTCOME 4 — USE EQUATIONS IN CALCULATING MEASUREMENTS OF PERIMETER, AREAS AND VOLUME

CONTENTS:

1. Understand and calculate Perimeters
2. Use equations in calculating areas of different plane figures.
3. Apply mathematical equations in computing volume and capacity.

ASSESSMENT CRITERIA:

1. Mathematical process in calculating perimeter, areas and volumes are used.
2. Calculation requirements in solving problems in perimeter, areas and volume are identified.
3. Mathematical equations are used in calculating perimeter, areas and volumes.

CONDITIONS:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

LEARNING ACTIVITIES

LEARNING OUTCOME: Use Equations in Calculating Measurements of Perimeter, Areas and Volume

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Solving Problems in Perimeter	Read Information Sheet No. 1.4-1 – Perimeter Answer Self Check 1.4-1 Compare your answer to the answer key.
Solving Problems in Areas	Read Information Sheet No. 1.4-2- Areas Answer Self Check 1.4-2 Compare your answer to the answer key.
Solving Problems in Volume	Read Information Sheet No. 1.4-3 – Volume Answer Self Check 1.4-3 Refer your answer to the answer key.

INFORMATION SHEET 1.4-1

Perimeter

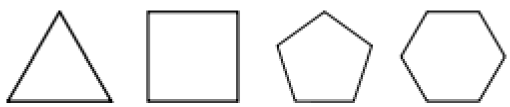
Learning Objectives:

After reading this Information Sheet, you should be able to use mathematical equations in solving problems in perimeter.

Perimeter

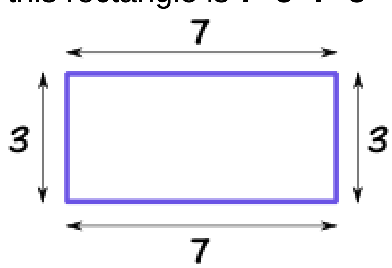
A **perimeter** is a path that surrounds a two-dimensional shape. The word comes from the Greek *peri* (around) and *meter*(measure). The term may be used either for the path or its length - it can be thought of as the length of the outline of a shape. The perimeter of a circle or ellipse is called its circumference.

Calculating the perimeter has considerable practical applications. The perimeter can be used to calculate the length of fence required to surround a yard or garden. The perimeter of a wheel (its circumference) describes how far it will roll in one revolution. Similarly, the amount of string wound around a spool is related to the spool's perimeter.

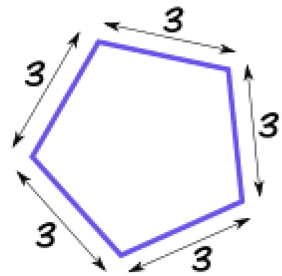


Perimeter is the distance around a two dimensional shape, or the measurement of the distance around something; the length of the boundary

Example 1: the perimeter of this rectangle is **7+3+7+3** = **20**

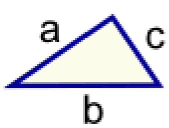


Example 2: the perimeter of this regular pentagon is **3+3+3+3+3** = **5×3 = 15**



Perimeter Formulas

1. **Triangle.** A plane figure having three sides and three angles.



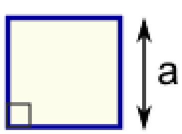
To solve for the Perimeter, add all given sides of the triangle.

P = a + b + c
where:
P = Perimeter
a and c = sides
b = base

Example: Find the perimeter of the triangle with sides 8m, 17m and 15m.

Perimeter = a + b + c
= 8m + 17m + 15
= 40m

2. **Square.** A square is a plane figure having four equal sides equal in lengths and four right angles.



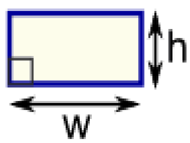
Perimeter = 4 × a

a = length of side

Example: Find the perimeter of the square with side 12m.

Perimeter = 4 x a (side)
= 4 x 12m
= 48m

3. **Rectangle.** A plane figure whose sides are parallel to each other having two equal sides and four right angles.

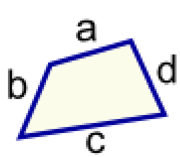


Perimeter = 2 × (w + h)
w = width
h = height

Example: Find the perimeter of the rectangle with length 2.5m and width 1.75m.

Perimeter = 2 x (w + h)
= 2 x (2.5m + 1.75m)
= 2 x (4.25m)
= 8.5m

4. **Quadrilateral.** A four sided plane figure with no two sides equal

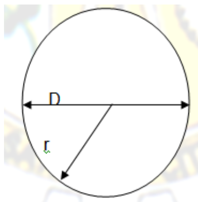


Quadrilateral Perimeter = a + b + c + d

Example: Find the perimeter of the quadrilateral with sides 3.6m, 1.2m,5.4m and 4.7m.

Perimeter = a + b + c + d
= 3.6m + 1.2m + 5.4m + 4.7m
= 14.9m

5. **Circle.** A plane figure with parts of which are equally distant from the center.



where:
D = diameter
r = radius
C = circumference
A = area

Diameter. Straight line through the center ending at the curve, dividing the circle into two parts. Equal to twice the radius.

$$D = 2r$$

Radius. Straight line from the center to the curve equals to ½ the diameter.

$$r = D/2$$

Circumference. The distance around the circle.

$$C = \pi D$$
where: $\pi = 3.1416$

The ratio of the circumference of any circle to its diameter is always about 3.14. This number is called π (the Greel letter pi). There is no decimal that is exactly equal to π, but approximately,

$$\pi = 3.14159265359$$

It is common to round π to 3.14.

Example: a. Find the circumference of a circle with radius 6 inches.

Circumference = $2\pi r$
= 2 (3.14) (6in)
= 37.68in

b. Find the circumference of a circle with a diameter of 1.6m.

circumference = πD
= 3.14 x 1.6m
= 5.02m

SELF-CHECK 1.4-1

Solve for the following:

1. How long will a fence be if the perimeter of a parallelogram has one side 54m and another side 28m?

2. Find the perimeter of triangular piece of garden lot with sides 4m, 8.3m and 6.1m.

3. The diameter of the tire of a bicycle is 34cm. how many turns would it take in order to travel 10,676cm?

4. Find the cost of fencing needed for the square field with one side 82ft if he cost of fencing is $2.75 per foot.

5. Find the perimeter of a basketball court 10m wide and 25m long.

ANSWER KEY 1.4-1

1. 164m
2. 18.4m
3. 100 turns
4. $902
5. 70m

INFORMATION SHEET 1.4-2

Areas

Learning Objectives:

After reading this Information Sheet, you should be able to use mathematical equations in solving problems in areas.

Measurement of Areas

As it often happens in everyday living, you find a requirement for knowing a formula to calculate the amount of material needed to complete a project or a particular structure. It also happens especially if you are in a workstation where you may need to know the measurement of a certain figure for you to decide on matters related to space.

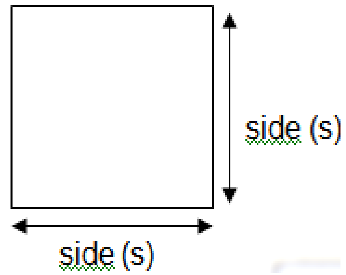
Plane figures are the most commonly encountered situations in the workshop. It has the measurement of two dimensions which refer to as the area.

Area is a quantity that expresses the extent of a two-dimensional surface or shape in a plane. It can be used to understand the amount of paint to buy for a wall or the cost of carpet for a room or the number of tiles needed for a floor. It is also calculated to know the dimensions of machines and equipment to be laid out in a certain work area.

The SI (system international) unit for measuring area is the square meter (m^2). It is defined as the area of a square whose sides measures one meter equally.

The following are plane figures with the corresponding mathematical formula in solving the area.

1. Square

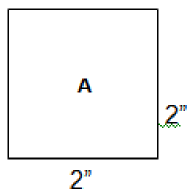


$A = s^2$

where: A = area

s = side

Example 1: Find the area of square A.



Area $= s^2$
$= 2^2$
Area $= 4in^2$

Example 2. Find out how many pieces of a 12in x 12in tile are needed to cover the square floor of a comfort room with side 60 inches.

First, you have to get the area of the tiles which is,

Area of a tile $= (12in)^2$
$= 144in^2$

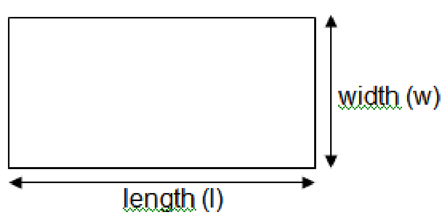
Then calculate the area of the floor to where the tiles will be put.

Area of floor $= (60in)^2$
$= 3600in^2$

So, area of the floor is $3600in^2$ and area of the tile is $144in^2$, to compute for the number of tiles to cover the floor of $3600in^2$, divide the area of the floor by the area of the tile.

Number of tiles needed $= \frac{3600in^2}{144in^2}$
$=$ 25 pcs

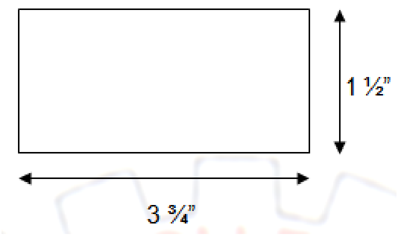
2. Rectangle



Area = l x w or l x h
where: l = length
w = width

h = height

Example 1. What is the area and perimeter of this rectangle of steel?



Area = l x w
= 3 ¾” x 1 ½”
= 15/4” x 3/2”
= 45/ 8
Area = 5 5/8in^2

Example 2. Find out how many liters of paint will be needed for wall of 4 meters high and 10 meters long if a liter of paint can cover 5m^2.

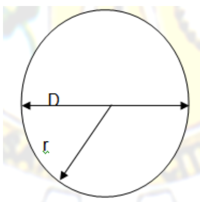
Area of the wall = l x h
= 10m x 4m
= 40m^2

1 liter of paint = 5m^2
So how many 5m^2 are there in 40m^2?

$$\frac{40m^2}{5m^2} = 8$$

There are eight 5m^2 in area of the wall, if one 5m^2 needs 1 liter of paint; it means 8 liters of paint will be needed.

3. **Circle**



$A = \pi r^2$ or $\frac{\pi D^2}{4}$

where: D = diameter
r = radius
π = 3.1416

Example 1. Find the area of the circle with radius 35cm.

$$\text{Area} = \pi r^2$$
$$= (3.14)\ (35\text{cm})^2$$
$$= (3.14)\ (1225\text{cm}^2)$$
$$= 3846.5\text{cm}^2$$

Example 2. The diameter of a wheel of a cart is 4.5 feet. Find the area of the wheel.

$$\text{Area} = \frac{\pi D^2}{4}$$

$$= \frac{(3.14)\ (4.5\text{ft})^2}{4}$$
$$= \frac{(3.14)\ (20.25\text{ft}^2)}{4}$$
$$= \frac{63.585\text{ft}^2}{4}$$
$$= 15.90\text{ft}^2$$

SELF-CHECK 1.4-2

Solve for the following:

1. A floor is 10ft by 15ft. What is the cost of tiling the floor if tile costs $10 per square foot?
2. The backyard of a new home is shaped like a trapezoid with a height of 45ft and base of 80ft and 110ft. Find the area of the backyard.
3. Find the area of a triangular flag 85cm high with a base 32cm.
4. How many hectares is a circular cabbage garden with a diameter of 1000 meters? (1hectare = 10,000 m^2)
5. A photograph is placed in a frame 24in by 30in. find the area of the photograph.

ANSWER KEY 1.4-2

1. $1,500.
2. 4,275 square feet
3. 1,360 cm^2
4. 78.5 hectares
5. 720 in^2

INFORMATION SHEET 1.4-3

Measurements of Volume

Learning Objectives:

After reading this Information Sheet, you should be able to identify solid figures and compute volumes of different solid figures.

Volume

The amount of metal contained in a block is measured in cubic units. The amount of free space in a hollow object is also measured in cubic units. The volume of a container is generally understood as the capacity of the container, for instance, the amount of fluid that a container could hold, rather than the amount of space the container itself displaces.

The SI unit for the measurement of volume is the cubic meter, abbreviated m^3. It is defined as the volume of a cube whose edges measures one meter and where the edges of the cube have the same lengths and 90^0 angles.

The volume of an object has the fundamental properties listed below.

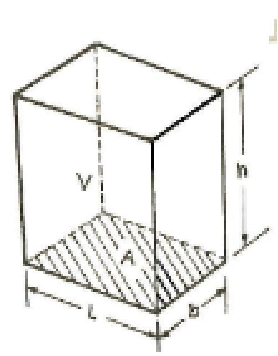
a. Every polyhedral region has a unique volume, dependent only on your unit cube.

b. A box has a volume of length x width x height (V= lwh).

c. Congruent figures have equivalent volume.

d. Total volume is the sum of all non-overlapping regions.

Formula for the Calculation of Volume



1. Regular Solids

$V = l \times b \times h$

$V = A \times h$

$A = l \times w$

where:

V = volume
A = Area of the Base
b = width
L = length

h = height

2. Prism

$V = A \times h$

where:

V = volume
A = Area
h = height of prism perpendicular to the base area

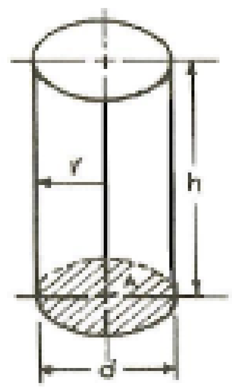
3. Cylinder

$V = A \times h$

$V = \frac{\pi d^2 \times h}{4}$

where:

V = volume
A = Area of the base
h = height
d = diameter
r = radius
π = pi (3.14.16)



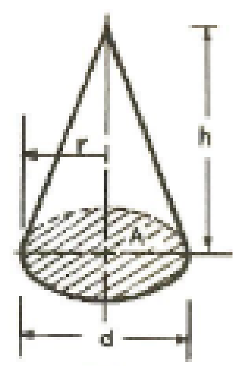
4. Cone

$V = \frac{A \times h}{3}$

$V = \frac{\pi d^2 \times h}{12}$

where:

V = volume
A = Area of the base
h = height
d = diameter
r = radius
π = pi (3.14.16)



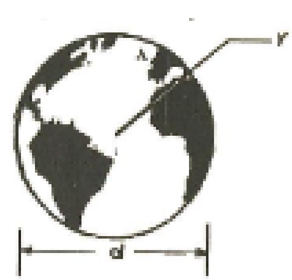
5. Sphere

$V = \frac{4\pi r^3}{3}$

$V = \frac{\pi d^3}{6}$

where:

V = volume
d = diameter
r = radius
π = pi (3.14.16)



SELF-CHECK 1.4-3

Use the given formulas above to solve the following problems.

1. A truck can carry a load of $30ft^3$. A piece of land is being excavated and the soil is being removed. If the excavation site is 45ft by 24ft by 9ft, how many full load of soil must be removed?
2. A cylindrical drum of gasoline is 7ft high and 4ft in diameter. How long will a gasoline last if $2ft^3$ are used each day?
3. How many cubic feet of water are there in a conical-shaped container 10 feet high and 14 feet across the base?
4. Find the volume of a coffee can with radius 6.3cm and height 15.8cm.
5. A sphere has a diameter of 96 inches. Find its volume.

ANSWER KEY 1.4-3

1. 324 truck loads
2. 43.96 days
3. 512.86 ft^3
4. 1,969.10cm^3
5. 463,011.84in^3

LEARNING OUTCOME 5 CALCULATE EQUIVALENTS MEASURES INVOLVING DIFFERENT SYSTEM OF MEASUREMENTS

CONTENTS:

1. Understand the differences between the different units of measurement
2. Calculate equivalent measures in English unit of measurement
3. Perform calculations in the conversion of Metric unit of measurement
4. Combine English and Metric unit of measurement and compute their approximate equivalents.

ASSESSMENT CRITERIA:

1. Calculation requirements in the conversion of different unit of measurement are identified.
2. Conversion of English unit of measurement is performed according to calculation requirement.
3. Conversion of Metric unit of measurement is performed according to calculation requirement.
4. Approximate equivalents of English and Metric units of measurement are computed

RESOURCES:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

LEARNING ACTIVITIES

LEARNING OUTCOME: Calculate Equivalents Measures Involving Different System of Measurements

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Familiarizing and Converting English Unit of Measurement	Read Information Sheet No. 1.5-1 on English Unit of Measurement Answer Self Check 1.5-1 Compare your answer to the answer key.
Familiarizing and Converting Metric Unit of Measurement	Read Information Sheet No. 1.5-2 on Metric Unit of Measurement Answer Self Check 1.5-2 Compare your answer to the answer key.
Converting English and Metric Equivalent	Read Information Sheet No. 1.5-3 on English and Metric Equivalent Answer Self Check 1.5-3 Refer your answer to the answer key.

INFORMATION SHEET 1.5-1

English Units of Measurements

Learning Objectives:

After reading this Information Sheet, you should be familiar with and able to convert English unit of measurement.

A. English System of Measurements

The English system or Imperial system also called as the inch-pound system of measure based on the use of the inch, pounds and the second as its units of length, mass, and time.

The English system, though in usage for some time, was found to contain many irregularities. A typical example was that the British foot is different in length from the American foot which was again different to the South Africa foot and so on. Because of this, an International system was conceived in favor of adopting a new system of weights and measures with the use of the Metric system.

Basic Units of Length in English Measurements

1 mile	=	1 760 yards (yd)
	=	5 280 feet (ft)
1 yard	=	3 feet (ft)
	=	36 inches (in)
1 foot	=	12 inches (in)

Basic Units of Weight in English Measurements

1 ton	=	2 000 pounds (lb)
1 pound (lb)	=	0.0005 ton
	=	16 oz
1 oz	=	0.0625 lb

Basic Units of Area in English Measurements

1 square foot (ft^2)	= 144 square inch (in^2)
1 square yard (yd^2)	= 9 square feet (ft^2)
	= 1296 square inch (in^2)
1 acre	= 43 560 ft^2
1 mile (mi)	= 640 acres

Basic Units of Volume in English Measurements

1 cubic yard (yd^3)	= 27 ft^3
	= 202 gallons

1 cubic feet (ft^3)	= 1 728 cubic inches (in^3)
	= 7.48 gallons (gal)
1 barrel	= 42 gallons (gal)
1 gallon	= 4 quart (qt)
1 bushel	= 32 quart (qt)
1 quart (qt)	= 57.75 cubic inch (in^3)

Conversion of Units in English Measurements

Examples:

1. How many feet are there in 6 yards?

Note: 1 yd = 3 feet

$$\frac{6\ \text{yd}}{1} \times \frac{3\ \text{feet}}{1\ \text{yd}} = 18\ \text{feet}$$

2. There are 12 inches in 1 foot, how many feet are there in 96 inches?

Note: 1 foot = 12 inches

$$\frac{96\ \text{inches}}{1} \times \frac{1\ \text{foot}}{12\ \text{inches}} = 8\ \text{feet}$$

3. Convert 12 pounds in oz.

Note: 1 lb = 16 oz

$$\frac{12\ \text{lb}}{1} \times \frac{16\ \text{oz}}{1\ \text{lb}} = 192\ \text{oz}$$

4. How many square ft are there in 3 square yards?

Note: 1 yd^2 = 9 ft^2

$$3\ \text{yd}^2 \times \frac{9\ \text{ft}^2}{1\ \text{yd}^2} = 27\ \text{ft}^2$$

5. If the computed volume of the object is 24 ft^3, what is the volume of the object in yd^3?

Note: 1 yd3 = 27 ft3

$$24\ \text{ft}^3 \times \frac{1\ \text{yd}^3}{27\ \text{ft}^3} = 0.88\ \text{yd}^3$$

SELF-CHECK NO. 1.5-1

Multiple Choice:

Direction: Analyze the problem carefully. Choose the correct answer and write the letter only in your answer sheet.

1. How many pounds are there in 60 kilograms?
    a. 130 lbs
    b. 132.3 lbs
    c. 133.5 lbs
    d. 135 lbs

2. If there are 16 oz in 1 pounds (lb) and 2.205 pounds (lb) in 1 kilogram, what is the equivalent value of 96 oz in kilogram?
    a. 1.85 kgs
    b. 1.93 kgs
    c. 2.50 kgs
    d. 2.72 kgs

3. The units of length, mass, and time for the Imperial system of measurements are,
    a. inch, pounds, seconds
    b. meter, pounds, seconds
    c. inch, kilogram, seconds
    d. meter, kilogram, seconds

4. There are 6 units of steel casements to be fabricated. Three units of these has a dimension of 4ft x 4ft., two units measure 4ft x 6ft., and other unit is 2 ft x 6 ft. What is the total area of the steel casement to be fabricated?
    a. 48 ft^2
    b. 60 ft^2
    c. 72 ft^2
    d. 108 ft^2

5. A cylindrical pail with a diameter of 12 inches and height of 15 inches contained of QDE paint. What is the volume of the pail?
    a. 1.0 ft^3
    b. 1.25 ft^3
    c. 1.5 ft^3
    d. 1.75 ft^3

ANSWER KEY 1.5-1

1. b
2. d
3. a
4. d
5. a

INFORMATION SHEET 1.5-2

Metric Units of Measurements

Learning Objectives:

After reading this Information Sheet, you should be familiar with and convert Metric unit of measurements.

Metric System of Measurement

The Metric System was born out of a revolution, the French Revolution. In 1790, during the height of the revolution, the Republican Convention hurriedly formed a Metric Committee composed of French scientist to develop the Metric Systems of Weighs and Measures based entirely on the Decimal Numbering System. The Committee enforced the use of the new system even for the measurement of time.

The extension of the Metric System is the SI units. The SI is an abbreviation meaning Systems International d'Unites in French or the International System of Units in English. The standard for world usage of the SI system has been established by the International Organization for Standardization (ISO).

Metric system is composed of units having uniform scale of relationships based on decimals. Its basic principle is the meter with scales of multiples and sub-multiples of ten. All units of surface area, capacity, volume, and weight are derived directly from the standard meter.

The meter is the basic SI units for all measurements of lengths. Quite a number of measuring instruments have been developed and graduated using the standard meter. It is most commonly used unit in the metric system where other units are derived

Basic Units of Length in Metric Measurement

1 kilometers (km)	= 10 hectometers (hm)
	= 100 decameters (dam)
	= 1000 meters (m)
1 meter	= 10 decimeters (dm)
	= 100 centimeters (cm)
	= 1000 millimeters (mm)

Basic Units of Weight in Metric Measurement

1 kilogram (kg)	= 10 hectogram (hg)
	= 100 decagram (dkg)
	= 1000 gram (g)

1 gram (g) = 10 decigram (dg)
= 100 centigram (cg)
= 1000 milligram (mg)

Basic Units of Area in Metric Measurement

1 square meter (m^2)= 10 000 square centimeter (cm^2)
1 hectare = 10 000 m^2
1 square kilometer (km^2) = 100 hectare

Basic Units of Volume in Metric Measurement

1 cubic meter cm^3 = 1000 liter (L)
1 L = 1000 milliliter (mL)
1 mL = 1 cm^3

Conversion of Units in Metric Measurements

Examples:

1. Convert 2.20 meters to centimeters.

Note: 1 meter = 100 cm

$$\frac{2.20\ \cancel{m}}{1} \times \frac{100\ cm}{1\ \cancel{m}} = 220\ cm$$

2. How many millimeters are there in 2.0 m?

Note: 1 m = 100 cm
1 cm =10 mm

$$\frac{2.0\ \cancel{m}}{1} \times \frac{100\ \cancel{cm}}{1\ \cancel{m}} \times \frac{10\ cm}{1\ \cancel{cm}} = 2000\ cm$$

3. If there are 1000 grams in 1 kilogram, how many grams are there in 3.75 kilogram?

Note: 1 kg = 1000grams

$$\frac{3.75\ \cancel{kg}}{1} \times \frac{1000\ g}{1\ \cancel{kg}} = 3\ 750\ grams$$

1. How many square meters are there in 2 units of steel window measures 120 cm x 150 cm?

Note: 1 m2 = 10 000 cm2

Area = 120 cm x 150 cm
= 18 000 cm2

2 units of Steel Window = 2 x 18 000 cm^2
= 36 000 cm^2

$$36\,000\ \cancel{cm^2} \times \frac{m^2}{10\,000\ \cancel{cm^2}} = 3.6\ m2$$

2. If a pail consists of 4 liters, how many cm^3 are there in a pail?

Note: 1 L = 1000 mL
1 mL = 1 cm^3

$$4\ \cancel{L} \times \frac{1000\ \cancel{mL}}{1\ \cancel{L}} \times \frac{cm^3}{1\ \cancel{mL}} = 4000\ cm^3$$

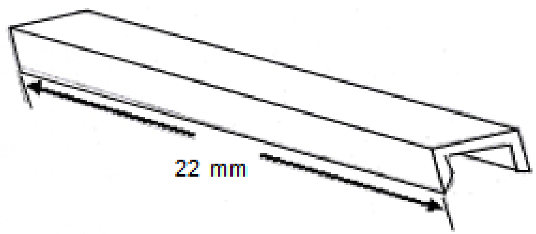
SELF-CHECK 1.5-2

Multiple Choice:

Direction: Analyze the problem carefully. Choose the correct answer and write the letter only in your answer sheet.

1. This piece of steel channel has a length of 22 millimeters. Express this measurement in centimeters.

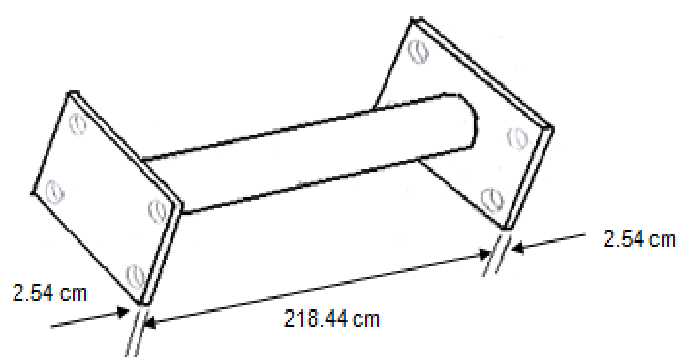
   a. 0.22 cm
   b. 2.02 cm
   c. 2.20 cm
   d. 2.22 cm



2. The basic of all SI measurement of length.
   a. meter
   b. millimeter
   c. centimeter
   d. kilogram

3. The basic units of length, mass, and time for the metric system of measurements are,
   a. inch, pounds, seconds
   b. inch, kilogram, seconds
   c. meter, pounds, seconds
   d. meter, kilogram, seconds

4. A pipe with end plate is shown.



A. Find the length of the pipe section in the weldment in millimeters.
   a. 2 184.40 mm

b. 2 184.44 mm
c. 21 844 mm
d. 21 844.4 mm

B. Find the thickness of one plate in millimeters.

a. 2.54 mm
b. 2.45 mm
c. 25.4 mm
d. 254 mm

C. Find the overall length in meters.

a. 2.23 m
b. 2.35 m
c. 2.55 m
d. 2.25 m

ANSWER KEY 1.5-2

1. c
2. a
3. d
4. A. a
   B. c
   C. a

INFORMATION SHEET NO. 1.5-3

English-Metric Equivalents

Learning Objectives:

After reading this Information Sheet, you should be able to perform calculations to convert English unit of measurements to Metric unit of measurement and vise versa.

English – Metric Equivalents

Most of the jobs require that you work in either English units or Metric, but not both. It is necessary, however, to occasionally convert units from one system to another.

Basic Units of English-Metric Measurement on Length and Weight

1 kilometer (km)	=	0.62137 mile (mi)
1 meter (m)	=	1.09361 yards (yd)
	=	3.28084 feet (ft)
	=	39.37 inches (in)
1 centimeter (cm)	=	0.39370 inch (in)
1 millimeter (mm)	=	0.039370 inch (in)
1 mile (mi)	=	1.609 kilometer (km)
1 yard (yd)	=	0.9144 meter (m)
1 foot (ft)	=	0.3048 meter (m)
1 inch	=	2.54 centimeter (cm)
	=	25.4 millimeter (mm)

Basic Units of English-Metric Measurement on Weight

1 kilogram (kg)	=	2.205 pounds (lb)
1000 kilograms (kgs)	=	1.102 tons
1.102 tons	=	2 204.621 pounds (lb)

Conversion of Units of English-Metric Equivalents

Example:

1. Convert the 3.0 meters to feet.

   Note: 1 m = 3.28084 ft

$$\frac{3.0\ \text{m}}{1} \times \frac{3.28084\ \text{ft}}{1\ \text{m}} = 9.84252\ \text{ft}$$

2. How many centimeters are there in 3 feet?

   Note: 1 inch = 2.54 cm
   1 ft = 12 inches

$$\frac{3\ \text{ft}}{1} \times \frac{12\ \text{in}}{1\ \text{ft}} \times \frac{2.54\ \text{cm}}{1\ \text{in}} = 91.44\ \text{cm}$$

3. How many inches are there in 1.20 meter?

   Note: 1 m = 39.37 in

$$\frac{1.20\ \text{m}}{1} \times \frac{39.37\ \text{in}}{1\ \text{m}} = 47.244\ \text{in}$$

4. If there are 2.54 cm in 1 inch, how many millimeters are there in 12 inches?

   Note: 1 in = 2.54 cm
   1 cm = 10 mm

$$\frac{12\ \text{in}}{1} \times \frac{2.54\ \text{cm}}{1\ \text{in}} \times \frac{10\ \text{mm}}{1\text{cm}} = 304.8\ \text{mm}$$

5. Convert 2.5 kilograms in pounds?

   Note: 1 kg = 2.205 lb

$$\frac{2.5\ \text{kg}}{1} \times \frac{2.205\ \text{lb}}{1\ \text{kg}} = 5.5125\ \text{lb}$$

6. What is the equivalent value of 1 kilogram to ounce?

   Note: 1 kg = 2.205 lb
   1 lb = 16 oz

$$\frac{1\ \text{kg}}{1} \times \frac{2.205\ \text{lb}}{1\ \text{kg}} \times \frac{16\ \text{oz}}{1\ \text{lb}} = 35.28\ \text{oz}$$

SELF-CHECK 1.5-3

Direction: Solve the following practical problems.

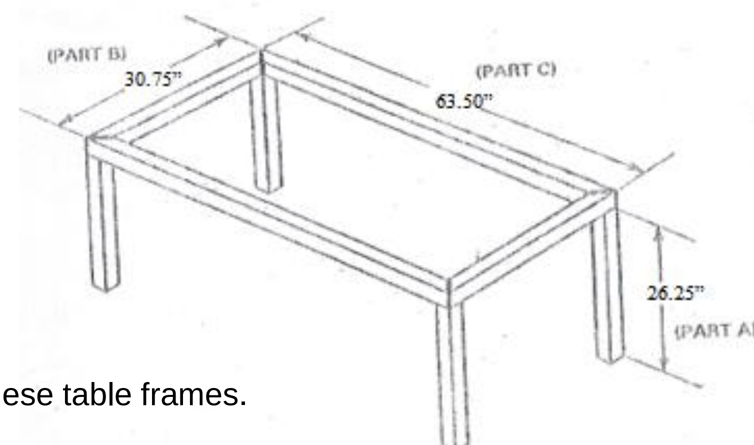
1. A 5 miles fun run is held to promote healthy heart to the public. If the average step of the participants is 0.8 m, how many steps do the participants will take in running the race?

2. There are 3 units of steel casements to be fabricated in the workshop. If each unit measures 4 ft x 6 ft, what is the total area of the project to be fabricated?

3. Convert the following measurements:

   a. 2250 m = ___________ ft.
   b. 3.5 kg = ___________ oz.
   c. 12 m^3 = ___________ ft^3

4. Use this illustration to compute the estimated materials for each part.



A welder makes 20 of these table frames.

a. How many centimeters of square steel tubing are required to complete the order for Part A?

b. How many centimeters of square steel tubing are required to complete the order for Part B?

c. How many meters of square tubing are required to complete the order for part C?

d. How many pieces of steel tubing are needed to fabricate 20 units of table frame if the standard length of the material to be used per piece is 6 meters?

ANSWER KEY 1.5-3

1. 10 056.25 steps

2. 6.68836608 m^3

3. a. 7 382.25 ft
   b. 123.48 oz
   c. 423.8380445 ft^3

4. a. 5 334 cm
   b. 3 124.2 cm
   c. 64.51285177 m
   d. 25 pieces

LEARNING OUTCOME 6 INTERPRET WORKPLACE DATA PRESENTED IN GRAPHS, CHARTS AND TABLES

CONTENTS:

1. Understand the uses of graphs and charts
2. Present workplace data in graphs and charts.
3. Interpret data presented in graphs and charts.

ASSESSMENT CRITERIA:

1. Uses of each graph and chart are identified.
2. Appropriate graphs and charts are constructed for a given data.
3. Data presented in graphs and charts are interpreted.

CONDITIONS:

Students/trainees must be provided with the following:

- Learning materials
- Activity sheets
- Reference materials

LEARNING ACTIVITIES

Learning Outcome: Interpret Workplace Data Presented In Graphs, Charts and Tables

LEARNING ACTIVITIES	SPECIAL INSTRUCTIONS
Constructing and Interpreting Graphs and Charts	Read Information Sheet No. 6.1 on Graphs and Charts Answer Self Check 6.1 Compare your answer to the answer key. Under Job Sheet 6.1, perform activity sheets 6.1-1, 6.1-2 and 6.1-3 Review the Performance through Performance Criteria Checklist

INFORMATION SHEET 1.6-1

Graphs and Charts

Learning Objectives:

After reading this Information Sheet, you should be able to construct and interpret graphs and charts.

"A picture is worth a thousand words." This is certainly true when you're presenting and explaining data. You can provide tables setting out the figures, and you can talk about numbers, percentages, and relationships forever. However, the chances are that your point will be lost if you rely on these alone. Put up a graph or a chart, and suddenly everything you're saying makes sense!

Graphs or charts help people understand data quickly. Whether you want to make a comparison, show a relationship, or highlight a trend, they help your audience "see" what you are talking about.

The trouble is there are so many different types of charts and graphs that it's difficult to know which one to choose. Click on the chart option in your spreadsheet program and you're presented with many styles. They all look smart, but which one is appropriate for the data you've collected?

Can you use a bar graph to show a trend? Is a line graph appropriate for sales data? When do you use a pie chart? The spreadsheet will chart anything you tell it to, whether the end result makes sense or not. It just takes its orders and executes them!

To figure out what orders to give, you need to have a good understanding of the mechanics of charts, graphs and diagrams. We'll show you the basics using four very common graph types:

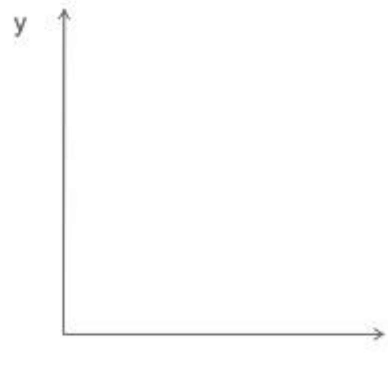
- Line graph
- Bar graph
- Pie chart
- Venn diagram

First we'll start with some basics.

X and Y Axes – Which is which?
To create most charts or graphs, excluding pie charts, you typically use data that is plotted in two dimensions, as shown in Figure 1.

- The horizontal dimension is the x-axis.
- The vertical dimension is the y-axis.

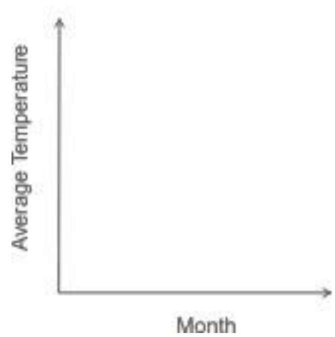
Figure 1: X and Y Axes



Tip:
To remember which axis is which, think of the x-axis as going **a**long the corridor and the y-axis as going **u**p the stairs – the letter "a" comes before "u" in the alphabet, just as "x" comes before "y."

When you come to plot data, the known value goes on the x-axis and the measured (or "unknown") value on the y-axis. For example, if you were to plot the measured average temperature for a number of months, you'd set up axes as shown in Figure 2:

Figure 2: The known value goes on the x axis and the measured value on the y axis



The next issue you face is deciding what type of graph to use.

Line Graphs
One of the most common graphs you will encounter is a line graph. Line graphs simply use a line to connect the data points that you plot. They are most useful for showing trends, and for identifying whether two variables relate to (or "correlate with") one another.

Trend data:

- How do sales vary from month to month?
- How does engine performance change as its temperature increases?

Correlation:

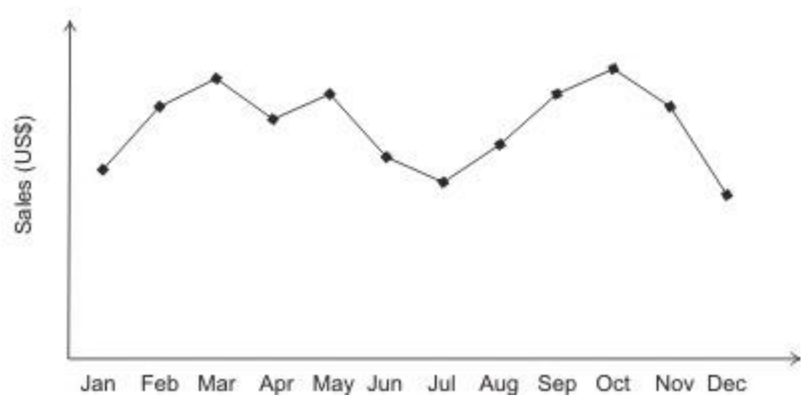
- On average, how much sleep do people get, based on their age?
- Does the distance a child lives from school affect how frequently he or she is late?

You can only use line graphs when the variable plotted along the x-axis is continuous – for example, time, temperature or distance.

Note:
When the y-axis indicates a quantity or percent and the x-axis represents units of time, the line graph is often referred to as a time series graph.

Example:
ABC Enterprises' sales vary throughout the year. By plotting sales figures on a line graph, as shown in Figure 3, it's easy to see the main fluctuations during the course of a year. Here, sales drop off during the summer months, and around New Year.

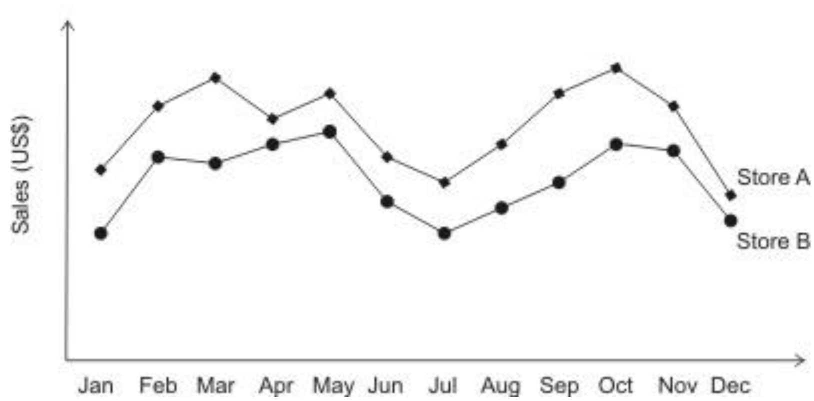
Figure 3: Example of a Line Graph



While some seasonal variation may be unavoidable in the line of business ABC Enterprises is in, it may be possible to boost cash flows during the low periods through marketing activity and special offers.

Line graphs can also depict multiple series. In this example you might have different trend lines for different product categories or store locations, as shown in Figure 4 below. It's easy to compare trends when they're represented on the same graph.

Figure 4: Example of a Line Graph with Multiple Data Series

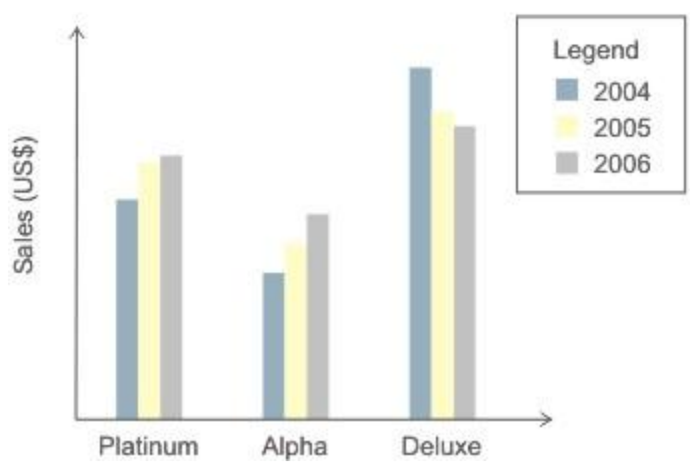


Bar Graphs

Another type of graph that shows relationships between different data series is the bar graph. Here the height of the bar represents the measured value or frequency: The higher or longer the bar, the greater the value.

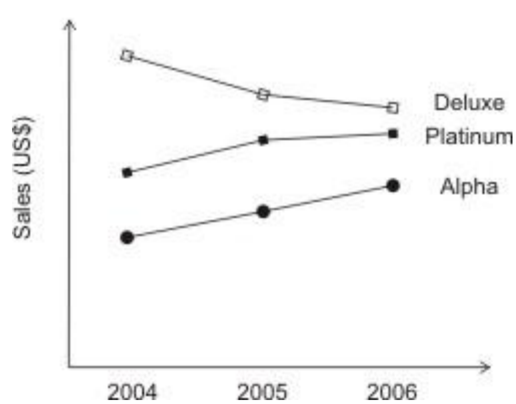
Example:
ABC Enterprises sells three different models of its main product, the Alpha, the Platinum, and the Deluxe. By plotting the sales each model over a three year period, it becomes easy to see trends that might be masked by a simple analysis of the figures themselves. In Figure 5, you can see that, although the Deluxe is the highest-selling of the three, its sales have dropped off over the three year period, while sales of the other two have continued to grow. Perhaps the Deluxe is becoming outdated and needs to be replaced with a new model? Or perhaps it's suffering from stiffer competition than the other two?

Figure 5: Example of a Bar Chart



Of course, you could also represent this data on a multiple series line graph as shown in Figure 6.

Figure 6: Data from Figure 5
Shown on a Line Graph



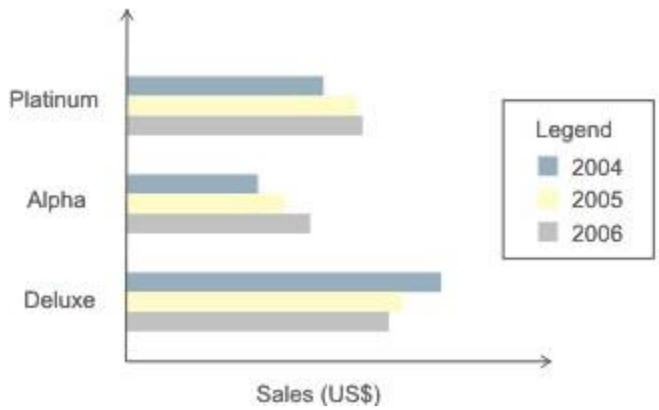
Often the choice comes down to how easy the trend is to spot. In this example the line graph actually works better than the bar graph, but this might not be the case if the chart had to show data for 20 models rather than just three. It's worth noting, though, that if you can use a line graph for your data you can often use a bar graph just as well.

The opposite is not always true. When your x-axis variables represent discontinuous data (such as different products or sales territories), you can only use a bar graph.

In general, line graphs are used to demonstrate data that is related on a continuous scale, whereas bar graphs are used to demonstrate discontinuous data.

Data can also be represented on a horizontal bar graph as shown in Figure 7. This is often the preferred method when you need more room to describe the measured variable. It can be written on the side of the graph rather than squashed underneath the x-axis.

Figure 7: Example of a Horizontal Bar Graph



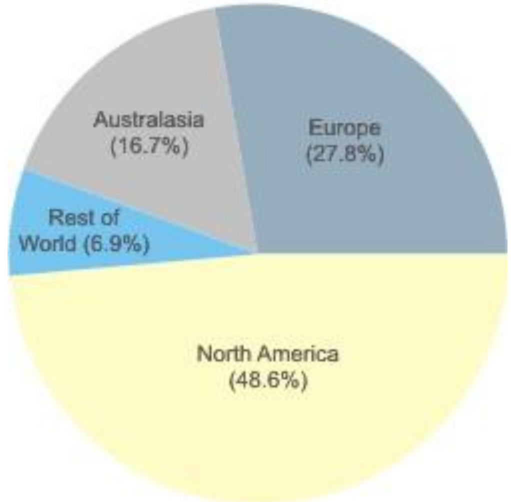
Note:
A bar graph is not the same as a histogram. On a histogram, the width of the bar varies according to the range of the x-axis variable (for example, 0-2, 3-10, 11-20, 20-40 and so on) and the area of the column indicates the frequency of the data. With a bar graph, it is only the height of the bar that matters.

Pie Charts
A pie chart compares parts to a whole. As such it shows a percentage distribution. The entire pie represents the total data set and each segment of the pie is a particular category within the whole.

So, to use a pie chart, the data you are measuring must depict a ratio or percentage relationship. You must always use the same unit of measure within a pie chart. Otherwise your numbers will mean nothing.

The pie chart in Figure 8 shows where ABC Enterprise's sales come from.

Figure 8: Example of a PieChart



Tip 1:

Be careful not to use too many segments in your pie chart. More than about six and it gets far too crowded. Here it is better to use a bar chart instead.

Tip 2:

If you want to emphasize one of the segments, you can detach it a bit from the main pie. This visual separation makes it stand out.

Tip 3:
For all their obvious usefulness, pie charts do have limitations, and can be misleading.

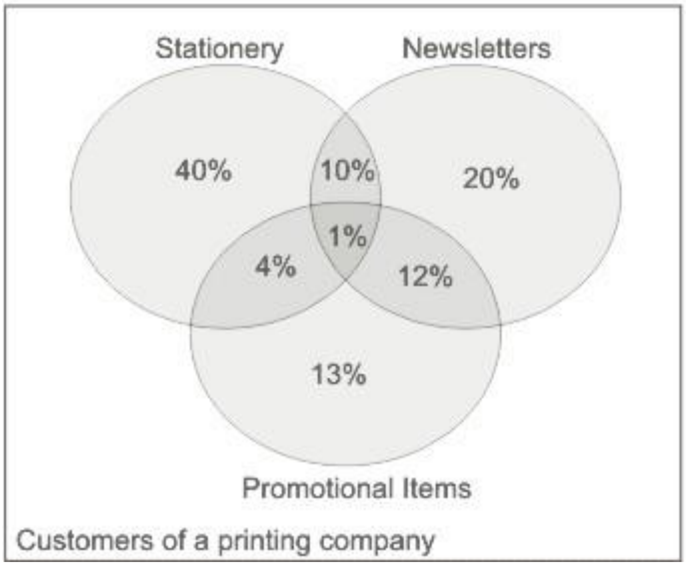
Venn Diagram

The last graph we will cover here is the Venn diagram. Devised by the mathematician John Venn in 1881, this is a diagram used to show overlaps between sets of data.

Each set is represented by a circle. The degree of overlap between the sets is depicted by the overlap between circles.

Figure 9 shows sales at Perfect Printing. There are three product lines: stationery printing, newsletter printing, and customized promotional items such as mugs.

Figure 9: An example of a Venn Diagram



By separating out the proportions of the business' customers that buy each type of product, it becomes clear that the majority of the biggest group of customers (55% of the total) – those who have their company stationery printed – are only using Perfect Printing for stationery. It's possible that they don't realize that Perfect Printing could also print their company newsletters and promotional items. As a result, Perfect Printing should consider designing some marketing activity to promote these product lines to its stationery customers.

Customers who get their newsletters printed by Perfect Printing, on the other hand, seem to be well aware that the company also offers stationery printing and promotional items.

A Venn diagram is a great choice to use when you are trying to convey the amount of commonality or difference between distinct groups.

SELF-CHECK 1.6-1

True or False: On the space provided before the number, write True if the statement is true and False if otherwise.

__________ 1. X and Y axis are used in plotting data in charts and graphs.

__________ 2. In plotting the data, the known value goes on the Y-axis and the measured value on the X-axis

__________ 3. Bar graphs are most useful for showing trends and for identifying which two variables relate or correlates with one another.

__________ 4. Line graphs are used to demonstrate data that is related on a continuous scale, whereas bar graphs are used to demonstrate discontinuous data.

__________ 5. A pie chart shows a percentage distribution of different categories within a total data set.

ANSWER KEY 1.6-1

1. True
2. False
3. False
4. True
5. True

JOB SHEET 1.6-1
Title: Contracting graphs and charts
Performance Objective: After completing this job, you should be able to construct different charts and graphs according to data interpretation requirement.
Materials: Compass, Protractor, Graphing paper, ruler, pencil and pen
Process/Procedure 1. Prepare all the materials needed in drawing graphs and charts. 2. Construct graphs and charts according to job requirements in activity sheets. 3. Interpret data presented in activity sheets. 4. Present your output to your trainer.

ACTIVITY SHEET 1.6-1

Title: Constructing Line graph

Materials: Graphing paper, ruler, pen and pencil

Procedure:

1. Go to the registrar's office and ask for the data on the number of enrollees for the last 5 years.
2. On a graphing paper, draw X and Y axes
3. Properly position variables according to standard procedures.
4. Plot the number of enrollees against each year.
5. Draw a line connecting the plotted points.
6. Interpret the graph drawn.

Interpretation must answer the following questions:

1. In which year were the least number of enrollees?
2. In which year were the most number of enrollees?
3. How many students were enrolled 5years ago?
4. What can you say about the trend of enrollment for the last five years? Increasing or decreasing?

ACTIVITY SHEET 1.6-2

Title: Constructing Bar graph

Materials: Graphing paper, ruler, pen and pencil

Data source: The number of students who passed the national assessment for different qualifications for the school year is as follows: 18 for baking NC ll, 23 for SMAW NC ll, 14 for Food Processing NC ll, 21 for Leather goods and 24 for Web page design.

Procedure:

1. From the above data source, construct a bar graph.
2. Properly position variables according to standard procedures.
3. Plot the number of students who passed the national assessment from the different qualification.
4. Interpret the graph drawn

Interpretation must answer the following questions:

1. In which qualification were the most number who passed the assessment?
2. In which qualification were the least number who passed the assessment?
3. In which of the baking NC ll and Web Page design NC ll had a higher number of passers?

ACTIVITY SHEET 1.6-3

Title: Constructing Pie Chart

Materials: Graphing paper, Protractor, ruler, pen and pencil

Procedure:

1. Draw Pie chart using the following data on Energy Producing Sources.

Sources	Percent
Petroleum	30%
Hydropower and Nuclear Power	10%
Coal	20%
Natural Gas	40%

2. Analyze the result.

PERFORMANCE CRITERIA CHECKLIST 1.6-1

Criteria Did i….	**YES**	**NO**
1. Plot the data in a line graph accurately?		
2. Put appropriate labels for the variables? (x and y-axes)		
3. Interpret data presented in the line graph?		
4. Draw bar graph according to the job requirement?		
5. Illustrate the difference between the variables in the bar graph?		
6. Interpret what is shown in graph?		
7. Draw the pie chart proportionately according to the data presented?		
8. Interpret the data presented in the pie chart?		

REVIEW OF COMPETENCY

Below is your performance criteria checklist for the module Use Basic Mathematical Concept

Assessment Performance Criteria	Yes	No
Mathematical process in solving problems in whole numbers are used.	☐	☐
Calculation requirements in solving problems in decimals are identified.	☐	☐
Four fundamental operations are used in solving whole numbers and decimals.	☐	☐
Mathematical process in solving problems in fractions are used.	☐	☐
Calculation requirements in solving problems in percentages are identified.	☐	☐
Four fundamental operations in solving problems in fractions are used.	☐	☐
Mathematical equations are used in solving problems in percentages.	☐	☐
The concepts about ratio and proportion are understood.	☐	☐
Calculation requirements in solving problems in ratio and proportions are identified.	☐	☐
Mathematical process in calculating perimeter, areas and volumes are used.	☐	☐
Calculation requirements in solving problems in perimeter, areas and volume are identified.	☐	☐
Mathematical equations are used in calculating perimeter, areas and volumes.	☐	☐
Calculation requirements in the conversion of different unit of measurement are identified.	☐	☐
Conversion of English unit of measurement is performed according to calculation requirement.	☐	☐
Conversion of Metric unit of measurement is performed according to calculation requirement.	☐	☐
Approximate equivalents of English and Metric units of measurement are computed.	☐	☐
Uses of each graph and chart are identified.	☐	☐
Data presented in graphs and charts are interpreted.	☐	☐

REFERENCES

1. CBLM in Welding , TESDA, Philippines

2. C. Ortiz Munsayac. Measurement in Metric with The SI System for High School and College Students. TRU-COPY Publishing House, Inc. Hiyas Homes Subdivision Pearl Cor. Sapphire Sts., Bo. Parada, Valenzuela, Metro Manila. Second Printing 1998.

3. Miller, Salzman Lial. Basic College Mathematics. Third Edition. HarperCollins Publishers Inc., 10 East 53rd Street, New York, NY 10022. 1991

4. http://www.mindtools.com

5. http://www.mathsisfun.com